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Simulations of Beam Emittance Growth from the Collective Relaxation of Space-Charge Nonuniformities

S. M. Lund, D. P. Grote, R. C. Davidson

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Steven M. Lund and David P. Grote
Lawrence Livermore National Laboratory (LLNL)
Ronald C. Davidson
Princeton Plasma Physics Laboratory (PPPL)

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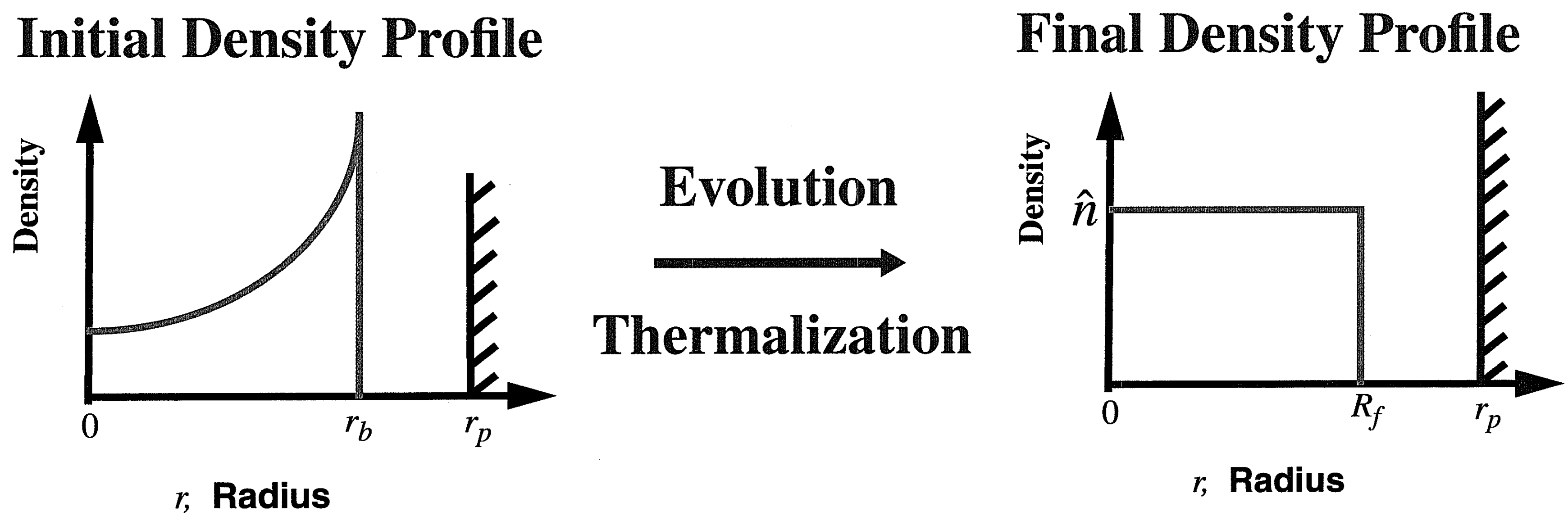
Abstract

Low-order models of In ideal linear focusing systems of space-charge-dominated beams,the transverse space-charge distribution of an ion beam tends to be nearly uniform within an elliptical envelope boundary. This produces linear transverse self-field forces within the beam that preserve beam phase space area (emittance). Non-ideal forces from aberrations of the applied focusing system and other sources can result in transverse density profiles that have strongly nonuniform charge density. This creates nonlinear self-field forces that can launch a broad spectrum of collective modes internal to the beam. There have been concerns that the free energy of such space-charge waves could lead to a loss of beam control and excessive emittance growth from oscillating nonlinear self-field forces. Here we employ the two-dimensional module of the WARP electrostatic particle in cell code to simulate this process. We find that collective relaxation processes tend to drive an initial nonuniform density beam to a final, relaxed state that is equilibrium-like with a more uniform, smoothed density profile and low-order residual oscillations. These relaxations appear driven by nonlinear wave interactions and phase-mixing associated with broad mode spectrums. This process is investigated for continuous focusing channels and periodic quadrupole focusing channels. It is found that surprising degrees of initial nonuniformity can be tolerated with modest emittance growth and that rms beam control can be maintained. Cases where the relaxation is fast and slow are analyzed. Simulation results are contrasted to earlier analytical theories[1] that should provide an upper bound on emittance growth if excessive halo is not generated. This work suggests that a surprising degree of initial space-charge nonuniformity can be tolerated in intense beams.

Approach

Assume that the spectrum of collective modes launched by initial space-charge nonuniformities is broad and will rapidly drive thermalization to a uniform density profile



Model the real transport channel (typically AG Quadrupole) with a continuous linear focusing channel

- Conservation of charge and energy can be used to connect the initial and final states
- Does not require understanding of the (possibly complicated) intervening evolution between the initial and final states

This energy method has been used effectively in many intense beam and nonneutral plasma studies. See, for example:

Martin Reiser, Theory and Design of Charged Particle Beams (Wiley, New York, 1994).

Ronald C. Davidson, An Introduction to the Physics of Nonneutral Plasmas (Addison-Wesley, New York, 1990)

Background

Significant beam space-charge nonuniformity has been observed in beams emerging from some intense heavy-ion injectors

- **Beams can have significant radial structure in the density profile (peaked, hollowed, etc.) depending on optical errors**
- **Such initial distributions are not well adapted to linear focusing channels**
- **Misadapted distribution can launch a spectrum of collective modes**

Collective modes can thermalize (relax) via phase mixing, nonlinear wave-wave interactions, etc.

- **Relaxation can result in a transfer of energy from the intense beam self-field to thermal energy leading to emittance growth**

Typical Heavy-Ion Fusion:

Potential Drop Across Beam
$$q(\phi|_{\text{beam center}} - \phi|_{\text{beam edge}}) \sim \frac{q\lambda}{4\pi\varepsilon_0} \sim 2.5\ \text{keV}$$

Spatial Average Particle Temp
$$\overline{T}_x \sim [\varepsilon_x^2/(2R^2)]E_b \sim 20\ \text{eV}$$

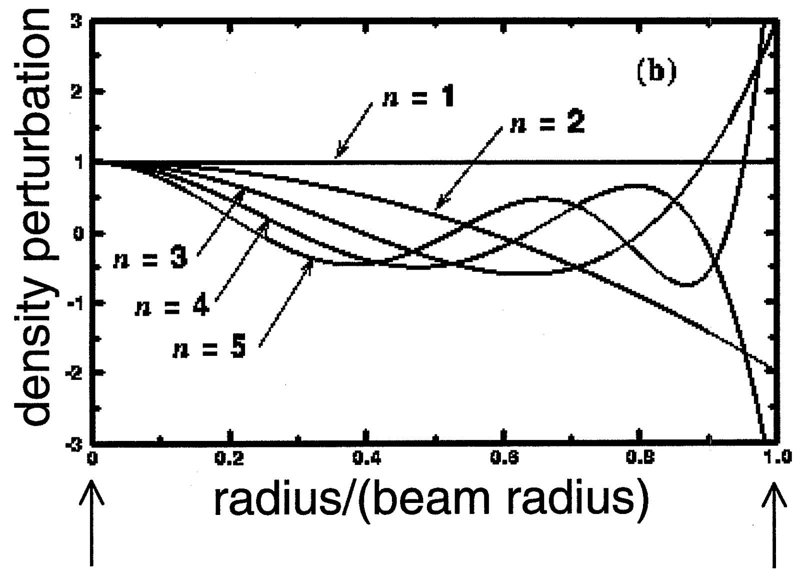
=> Even a small thermalization space-charge energies could result in large emittance increases

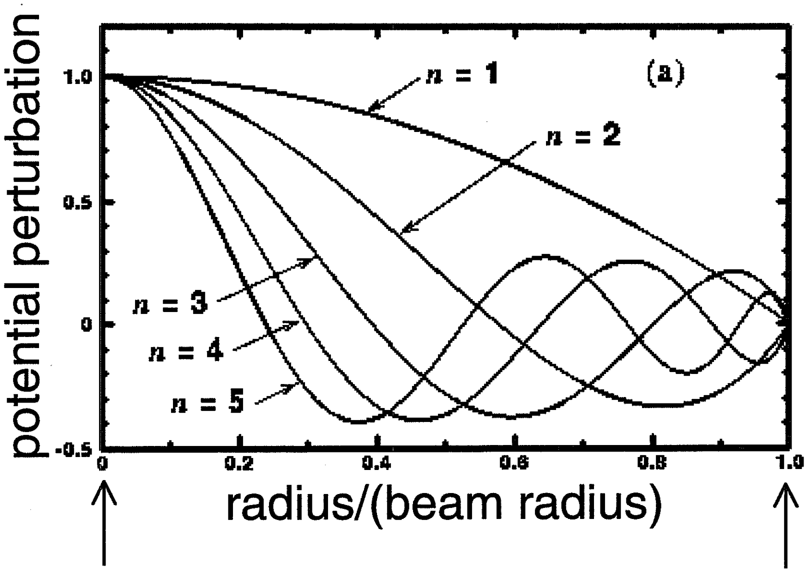
Can a better estimate be provided of possible emittance increases in space-charge dominated beams due to the thermalization of space-charge nonuniformities?

Initial distribution distortions will launch a spectrum of collective mode perturbations that evolve

Kinetic and fluid theories have been employed to analyze perturbations on a uniform density intense-beam equilibrium **[Lund and Davidson, Phys. Plasmas, 5 3028 (1998)]**

Small Amplitude Perturbations (arbitrary units, kinetic and fluid theory)





Mode Dispersion Relation (fast branch, from fluid theory)

$$\frac{\sigma_n}{\sigma_0} = \sqrt{2 + 2\left(\sigma / \sigma_0\right)^2 (2n^2 - 1)}$$

σ_n = mode phase advance

n = 1,2,3, ……

Example:

$\sigma_0 = 80^o$, $\sigma/\sigma_0 = 0.2$

$\sigma_1 = 115^o$, $\sigma_5 = 182^o$, ….

Theoretical Model(1)

Employ the conventional Vlasov-Poisson System to model the electrostatic evolution of a nonrealativistic (for convenience) beam of charged particles of mass m and charge q propagating with axial kinetic energy E_b in a continuous focusing channel

$$\left\{\frac{\partial}{\partial s}+\frac{\partial H}{\partial \vec{x}'}\bullet\frac{\partial}{\partial \vec{x}}-\frac{\partial H}{\partial \vec{x}}\bullet\frac{\partial}{\partial \vec{x}'}\right\}f(\vec{x},\vec{x}',s) = 0$$

$$H = \frac{\vec{x}'^2}{2}+\frac{k_{\beta 0}^2}{2}\vec{x}^2+\frac{q}{2E_b}\phi$$

$$\nabla^2\phi = -4\pi q\int f(\vec{x},\vec{x}',s)dx'dy'$$

$$\phi(\vec{x},s)\Big|_{r=r_p} = 0$$

Here s is the axial propagation distance (denote $x' = dx/ds$), $k_{\beta 0} = const$ is the applied focusing force, r_p is the radius of a perfectly conducing, cylindrical beam pipe, and $f(\vec{x},\vec{x}',s)$ is the single particle distribution function

Theoretical Model(2)

Any solution of the Vlasov-Poisson System will be consistent with the rms envelope equation:

$$\boxed{R'' + k_{\beta 0}^2 R - \frac{Q}{R} - \frac{\varepsilon_x^2}{R^3} = 0}$$

$$Q = \frac{q\lambda}{[4\pi\varepsilon_0]E_b} = \text{Perveance (constant)}$$

$$R = 2\langle x^2\rangle^{1/2} = \text{rms edge radius}$$

$$\varepsilon_x^2 = 16[\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle] = \text{rms edge emittance squared}$$

$$\langle\ldots\rangle = \frac{\iint \ldots f\, dx'dy'dxdy}{\iint f\, dx'dy'dxdy} = \text{transverse statistical average}$$

- **The emittance is a statistical measure of beam phase space area and evolves according to the full Vlasov-Poisson system**

Global Conservation Constraints

The Vlasov Poisson System has several global conservation constraints

Generalized Entropy (S any smooth function**):**

$$U_s = \int\int S(f)\,dx'dy'dxdy = \text{const}$$

Normalized Angular Momentum:

$$P_\theta = \int\int (xy' - yx')f\,dx'dy'dxdy = \text{const}$$

System Energy (per unit axial length**):**

$$\boxed{E_s = E_b\int\int \vec{x}'^2 f\,dx'dy'dxdy + E_b\int\int k_{\beta 0}^2 \vec{x}^2 f\,dx'dy'dxdy + \int \frac{|\nabla\phi|^2}{8\pi}dxdy = \text{const}}$$

A special case of the generalized entropy constraint with $S(f) = f$ **is line charge conservation:**

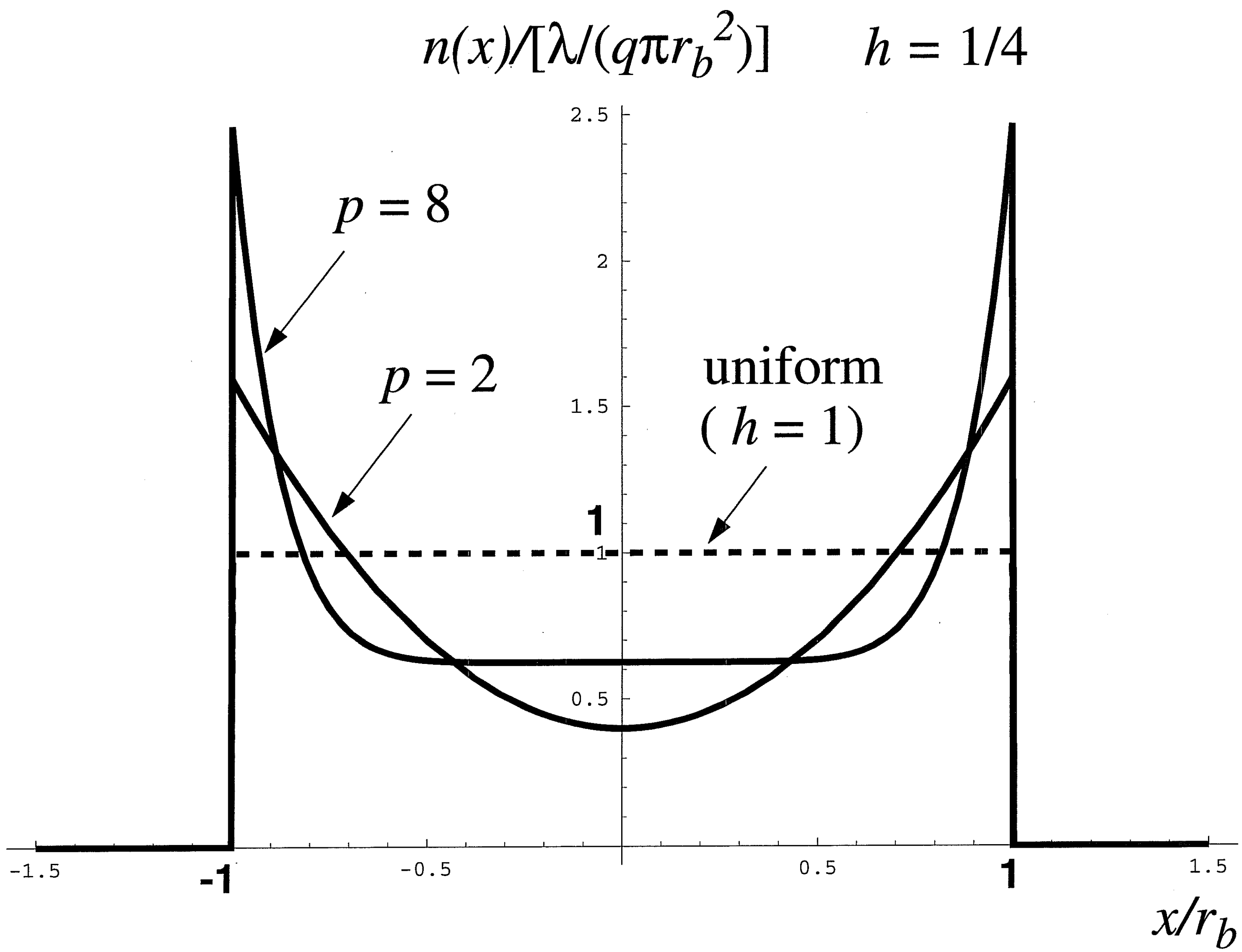
$$\boxed{\lambda = \int\int f\,dx'dy'dxdy = \text{const}}$$

System line charge and energy conservation will be employed here to analyze beam emittance changes on thermalization of space-charge nonuniformities.

Choice of Initial Distribution Parameterization (2)

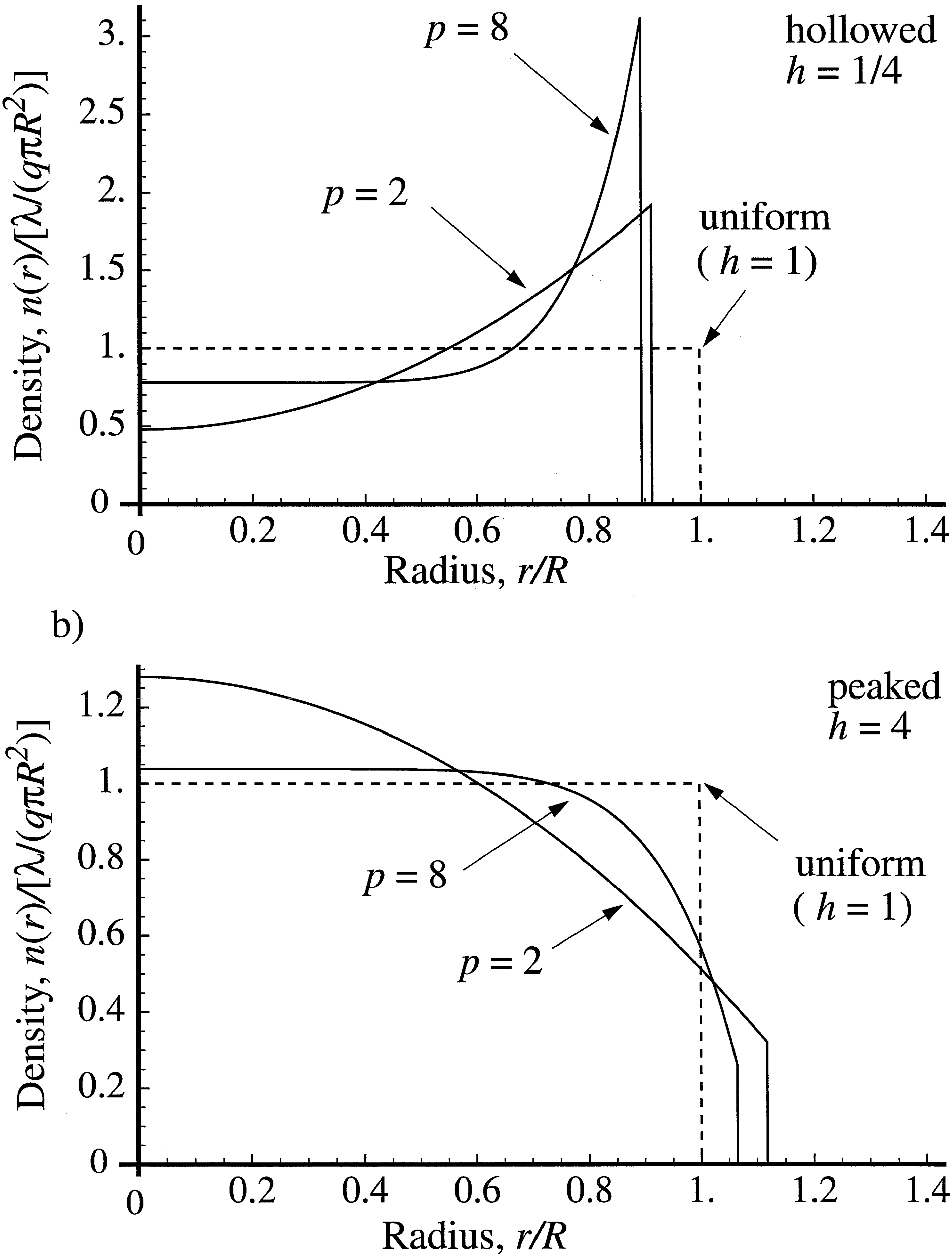
Density profile parameterization can model strong beam hollowing

- **Fix hollowing factor** $h = 1/2$ **and line charge** $\lambda = \text{const}$



a)

b)



hollowed
$h = 1/4$
$p = 8$
$p = 2$
uniform
($h = 1$)
Density, $n(r)/[\lambda/(q\pi R^2)]$
Radius, r/R
peaked
$h = 4$
$p = 8$
$p = 2$
uniform
($h = 1$)
Density, $n(r)/[\lambda/(q\pi R^2)]$
Radius, r/R

Connections of Initial to Final State

Assume that the final state is rms matched ($R_f' = 0 = R_f''$) with a uniform density profile ($h = 1$) and that charge ($\lambda = \mathrm{const}$) and system energy (E_s) is conserved

- **Energy constraint:**

$$\frac{(R_f/R_i)^2 - 1}{1 - (\sigma_i/\sigma_0)^2} + \frac{p(1-h)[p + 4 + (p+3)ph]}{(p+2)(p+4)(2+ph)^2} - \log\left[\sqrt{\frac{(p+2)(ph+4)}{(p+4)(ph+2)}}\frac{R_f}{R_i}\right] = \frac{E_b}{2q\lambda}(R_i R_i')'$$

-- **Here and henceforth, subscripts i and f refer to the initial and final beams**

-- **For an initially rms matched beam envelope $R_i' = 0 = R_i''$**

-- **Solve for the final to intial ratio of rms radii, R_f/R_i, in terms of the system parameters h, σ_i/σ_0, etc.**

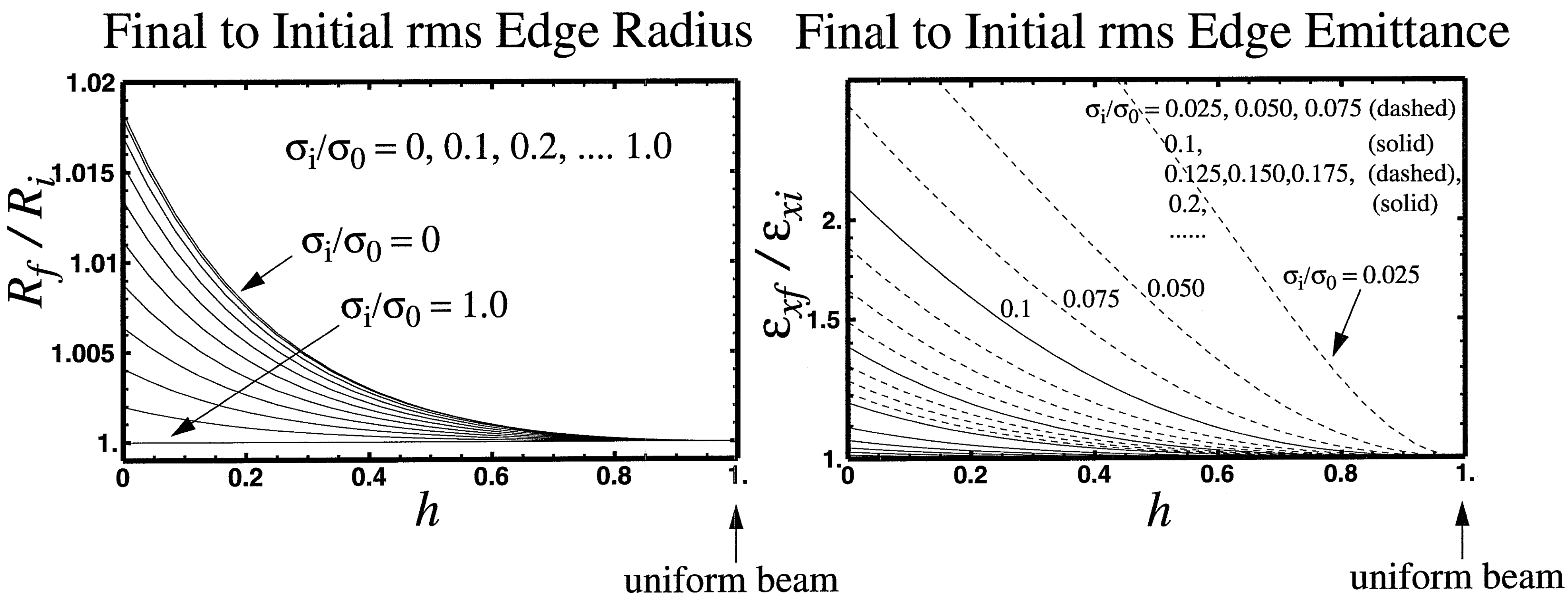
- **The ratio of final to initial emittance (ε_x) can be calculated (or similarly any other quantity in final to initial state ratio) as:**

$$\frac{\varepsilon_{xf}}{\varepsilon_{xi}} = \frac{R_f}{R_i}\sqrt{\frac{(R_f/R_i)^2 - \left[1 - (\sigma_i/\sigma_0)^2\right]}{(\sigma_i/\sigma_0)^2 - R_i''/(k_{\beta 0}^2 R_i)}}$$

-- **Analyze using constraint equation for R_f/R_i and system parameters**

Changes on Relaxation -- initially rms matched ($R_i' = 0 = R_i''$), hollowed ($p = 2,\ 0 < h < 1$) density profile

Assume full relaxation to a uniform, matched density profile and solve the equations of constraint to obtain:



Increase in beam rms radius (R) is very small

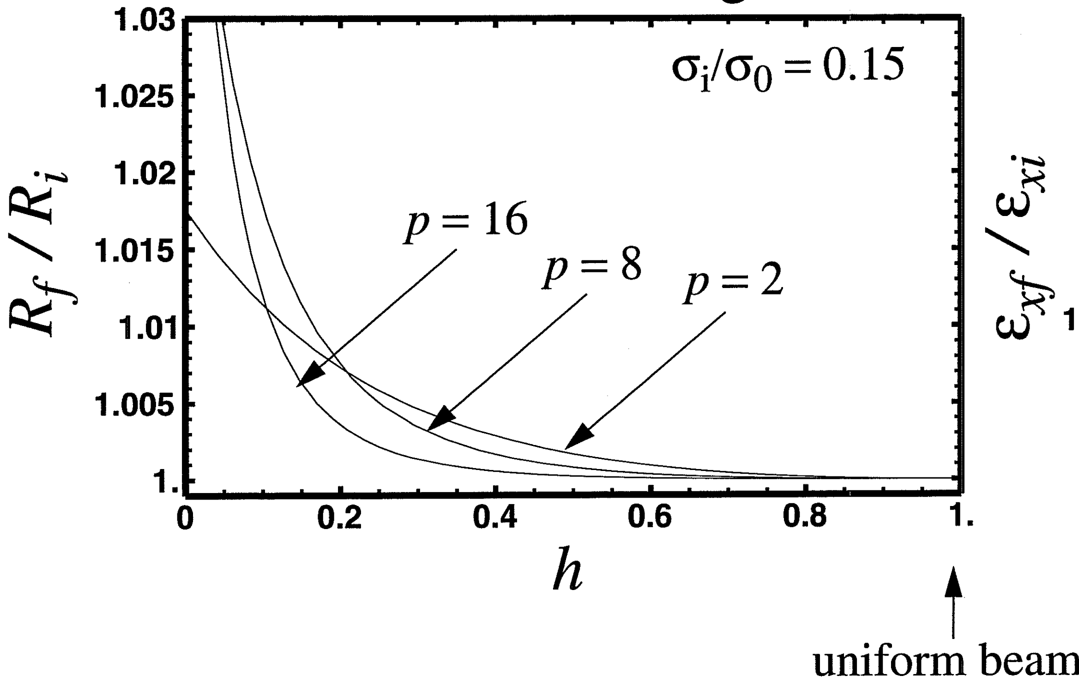
Emittance (ε_x) growth modest even for large hollowing factors ($h \to 0$)

Larger growth for stronger initial space-charge strength ($\sigma_i/\sigma_0 \to 0$)

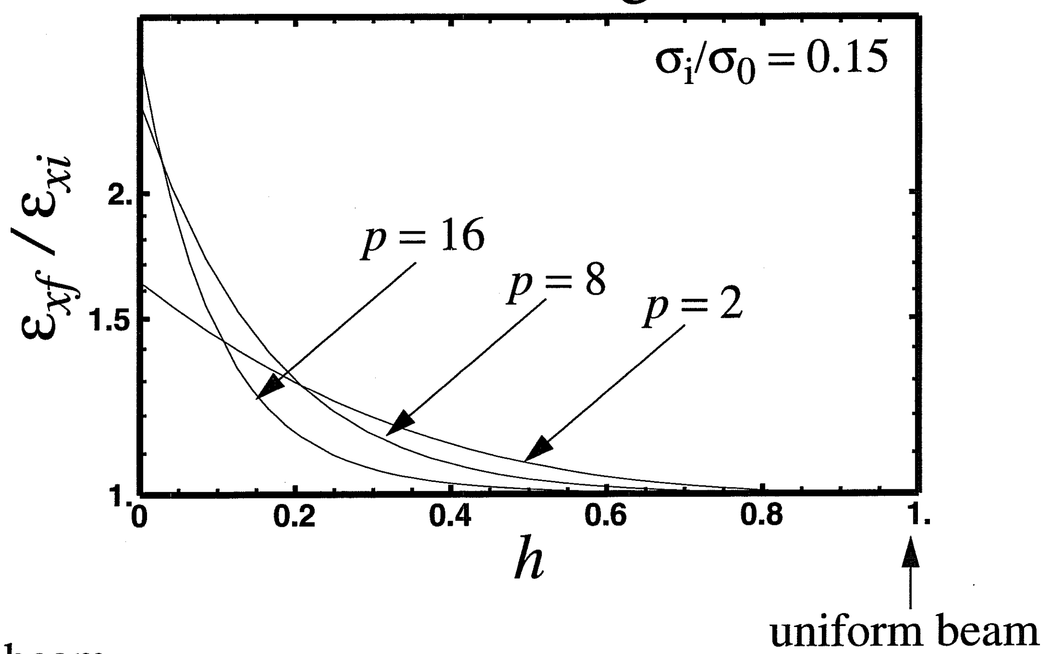
Changes on Relaxation -- initially rms matched ($R_i' = 0 = R_i''$), hollowed ($0 < h < 1$) density profile

Assume full relaxation to a uniform, matched density profile keeping fixed space-charge intensity ($\sigma/\sigma_0 = 0.15$) and vary the hollowing paramter h and the steepening parameter p to examine sensitivity in changes in rms beam parameters

Final to Initial rms Edge Radius



Final to Initial rms Edge Emittance

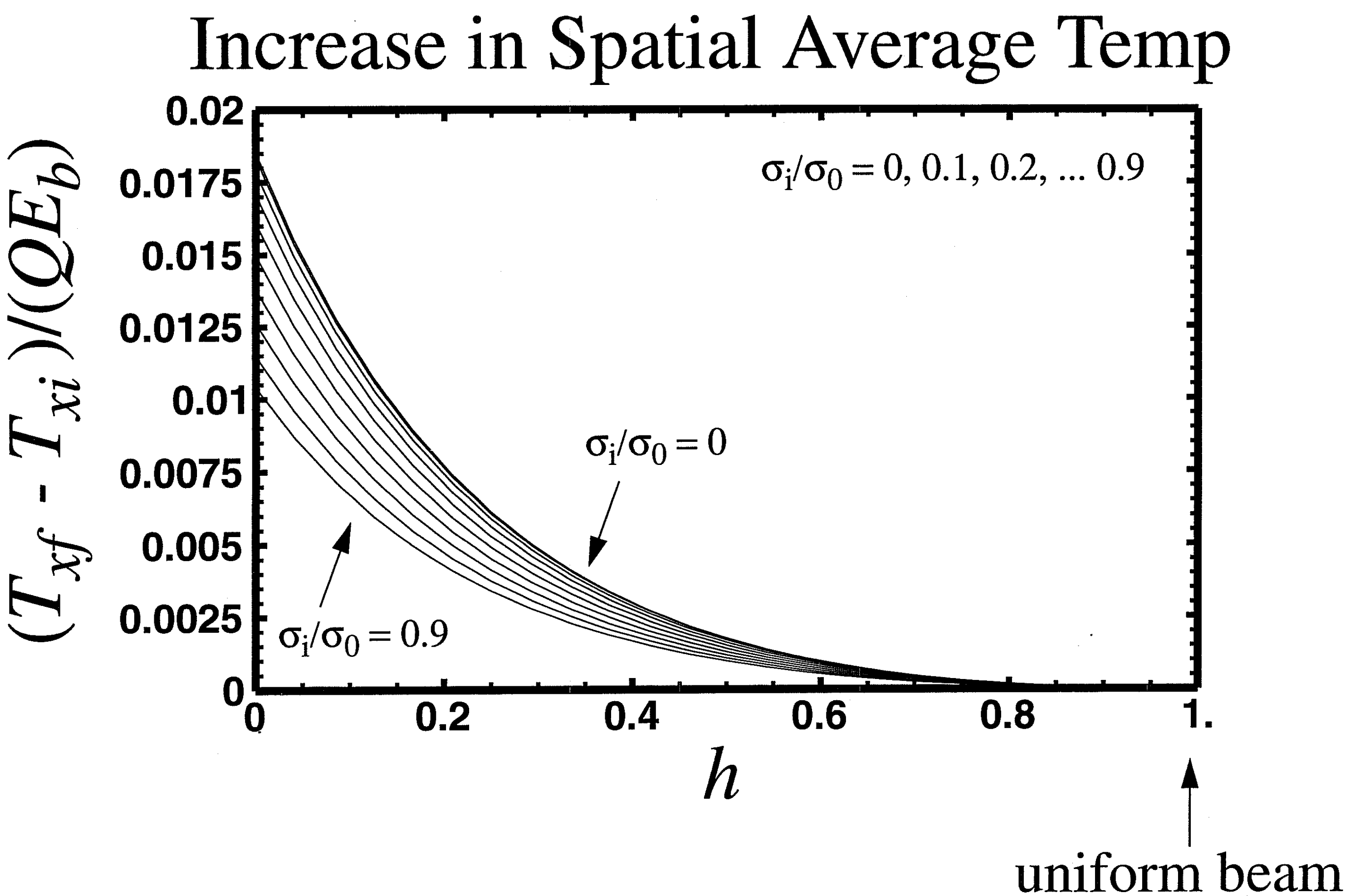


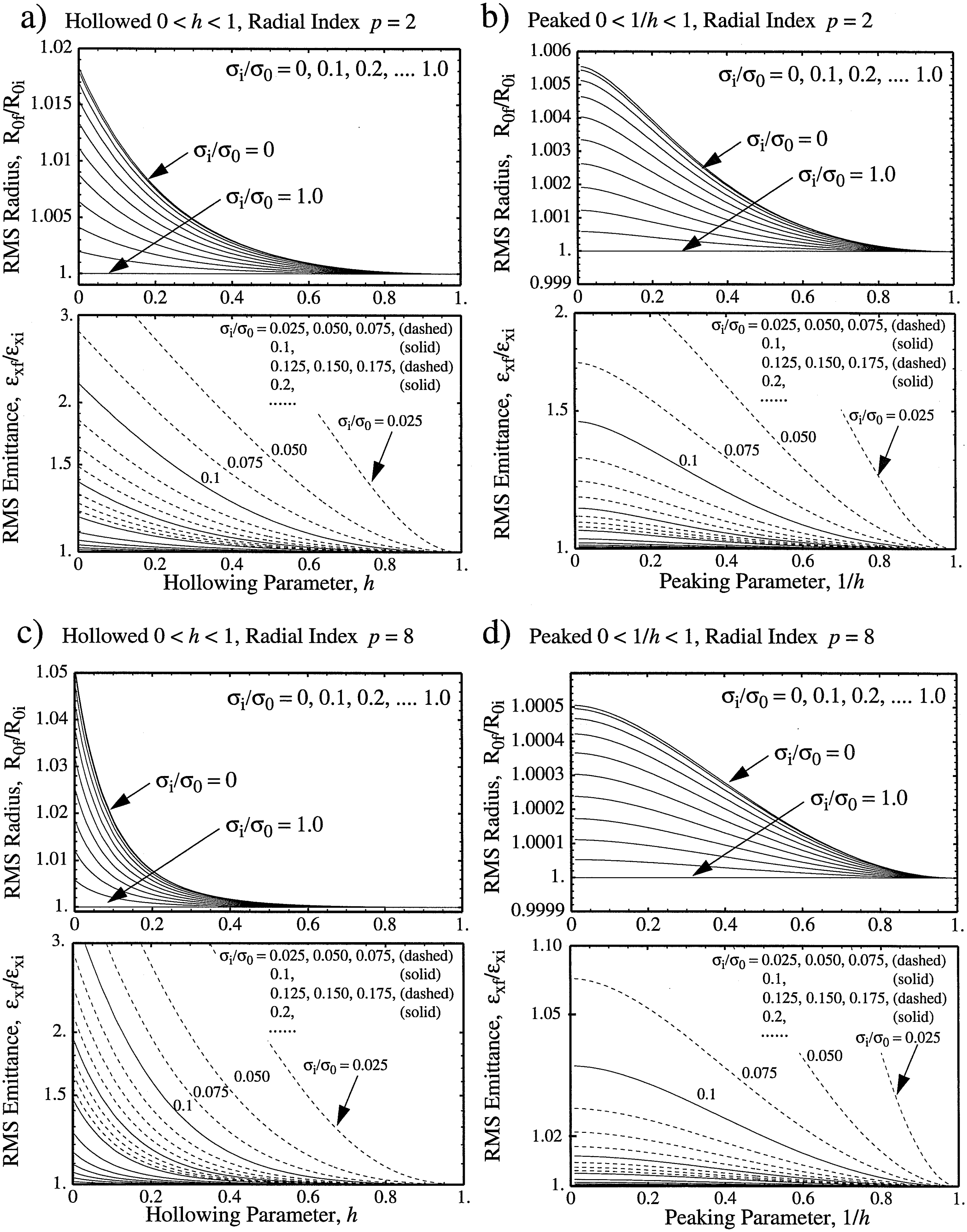
For all but the most exteme combination of steeping parameters ($p >> 1$) and hollowing factors ($h \to 0$) the growth in rms radius and emittance remains modest

Changes on Relaxation -- initially rms matched ($R_i' = 0 = R_i''$), hollowed ($p = 2,\ 0 < h < 1$) density profile

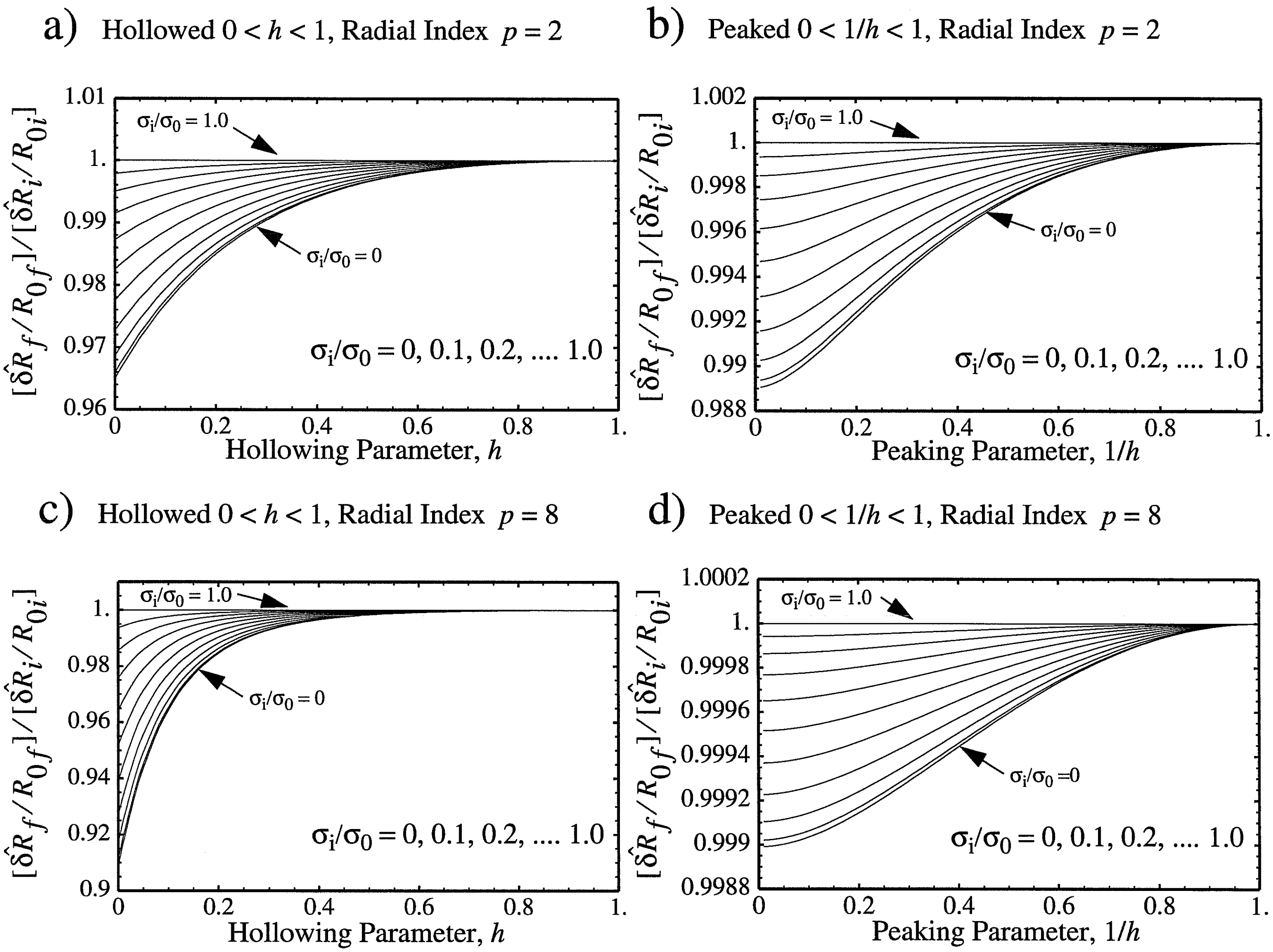
Recast results in terms of the increase in spatial average temperature on relaxation to a uniform density profile

$$\frac{\overline{T_x}}{\overline{E_b}} = \frac{\varepsilon_x^2}{2R^2}$$

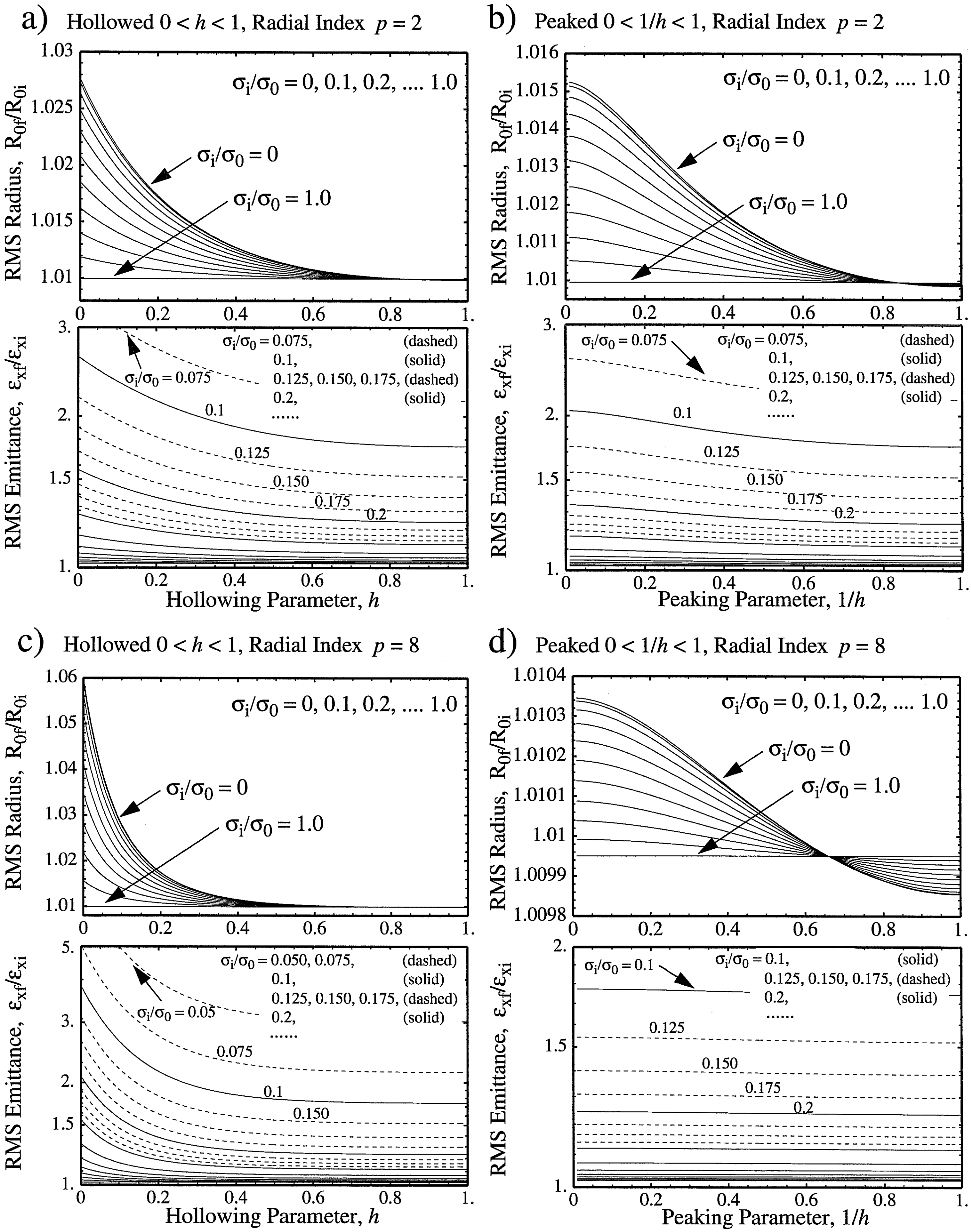




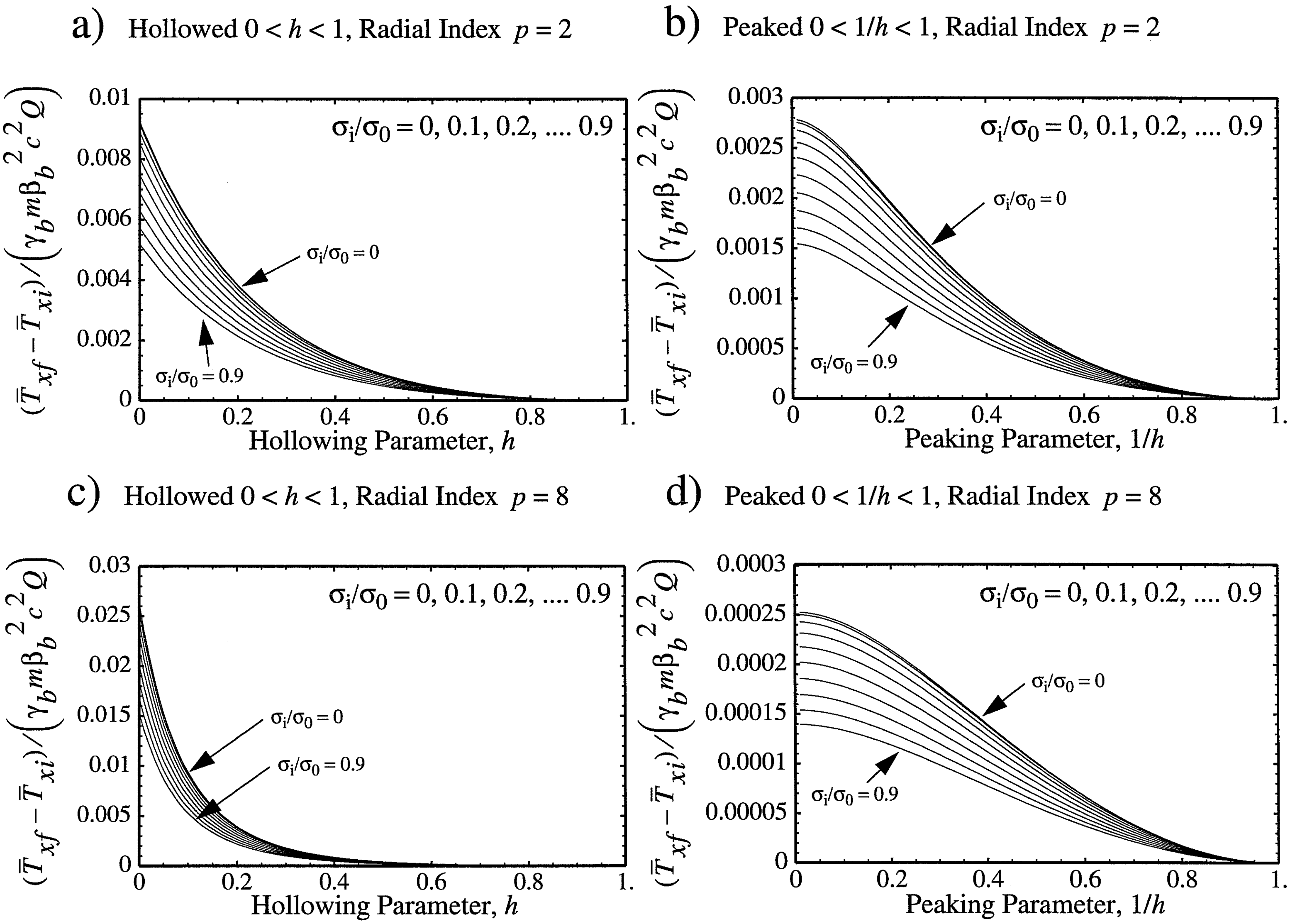
a) Hollowed 0 < h < 1, Radial Index p = 2
σi/σ0 = 0, 0.1, 0.2, ... 1.0
σi/σ0 = 0
σi/σ0 = 1.0
RMS Radius, R0f/R0i
σi/σ0 = 0.025, 0.050, 0.075, (dashed)
0.1, (solid)
0.125, 0.150, 0.175, (dashed)
0.2, (solid)
......
σi/σ0 = 0.025
0.050
0.075
0.1
RMS Emittance, εxf/εxi
Hollowing Parameter, h
b) Peaked 0 < 1/h < 1, Radial Index p = 2
σi/σ0 = 0, 0.1, 0.2, ... 1.0
σi/σ0 = 0
σi/σ0 = 1.0
RMS Radius, R0f/R0i
σi/σ0 = 0.025, 0.050, 0.075, (dashed)
0.1, (solid)
0.125, 0.150, 0.175, (dashed)
0.2, (solid)
......
σi/σ0 = 0.025
0.050
0.075
0.1
RMS Emittance, εxf/εxi
Peaking Parameter, 1/h
c) Hollowed 0 < h < 1, Radial Index p = 8
σi/σ0 = 0, 0.1, 0.2, ... 1.0
σi/σ0 = 0
σi/σ0 = 1.0
RMS Radius, R0f/R0i
σi/σ0 = 0.025, 0.050, 0.075, (dashed)
0.1, (solid)
0.125, 0.150, 0.175, (dashed)
0.2, (solid)
......
σi/σ0 = 0.025
0.050
0.075
0.1
RMS Emittance, εxf/εxi
Hollowing Parameter, h
d) Peaked 0 < 1/h < 1, Radial Index p = 8
σi/σ0 = 0, 0.1, 0.2, ... 1.0
σi/σ0 = 0
σi/σ0 = 1.0
RMS Radius, R0f/R0i
σi/σ0 = 0.025, 0.050, 0.075, (dashed)
0.1, (solid)
0.125, 0.150, 0.175, (dashed)
0.2, (solid)
......
σi/σ0 = 0.025
0.050
0.075
0.1
RMS Emittance, εxf/εxi
Peaking Parameter, 1/h



a) Hollowed 0 < h < 1, Radial Index p = 2
b) Peaked 0 < 1/h < 1, Radial Index p = 2
c) Hollowed 0 < h < 1, Radial Index p = 8
d) Peaked 0 < 1/h < 1, Radial Index p = 8
[δR̂f/R0f]/[δR̂i/R0i]
σi/σ0 = 1.0
σi/σ0 = 0
σi/σ0 = 0, 0.1, 0.2, ... 1.0
Hollowing Parameter, h
Peaking Parameter, 1/h



a) Hollowed 0 < h < 1, Radial Index p = 2
b) Peaked 0 < 1/h < 1, Radial Index p = 2
c) Hollowed 0 < h < 1, Radial Index p = 8
d) Peaked 0 < 1/h < 1, Radial Index p = 8
RMS Radius, R0f/R0i
RMS Emittance, εxf/εxi
σi/σ0 = 0, 0.1, 0.2, ... 1.0
σi/σ0 = 0
σi/σ0 = 1.0
σi/σ0 = 0.075, (dashed)
0.1, (solid)
0.125, 0.150, 0.175, (dashed)
0.2, (solid)
......
σi/σ0 = 0.075
σi/σ0 = 0.050, 0.075, (dashed)
σi/σ0 = 0.05
σi/σ0 = 0.1, (solid)
σi/σ0 = 0.1
0.075
0.1
0.125
0.150
0.175
0.2
Hollowing Parameter, h
Peaking Parameter, 1/h



a) Hollowed 0 < h < 1, Radial Index p = 2
b) Peaked 0 < 1/h < 1, Radial Index p = 2
c) Hollowed 0 < h < 1, Radial Index p = 8
d) Peaked 0 < 1/h < 1, Radial Index p = 8
$(\bar{T}_{xf} - \bar{T}_{xi})/(\gamma_b m \beta_b^2 c^2 Q)$
$\sigma_i/\sigma_0 = 0, 0.1, 0.2, ... 0.9$
$\sigma_i/\sigma_0 = 0$
$\sigma_i/\sigma_0 = 0.9$
Hollowing Parameter, h
Peaking Parameter, 1/h

Large Scale WARP xy Slice Simulations Investigate Collective Relaxation Processes

The electrostatic WARP code was developed by LLNL for simulation of intense ion beams for Heavy Ion Fusion applications.

- Extensive code with xy, r-z, 3D, and other modules
- Variety of field solvers, particle movers, and diagnostics to check modeling approximations
- Interpreter based (steerable) for flexability
- Serial and massively parallel simulations

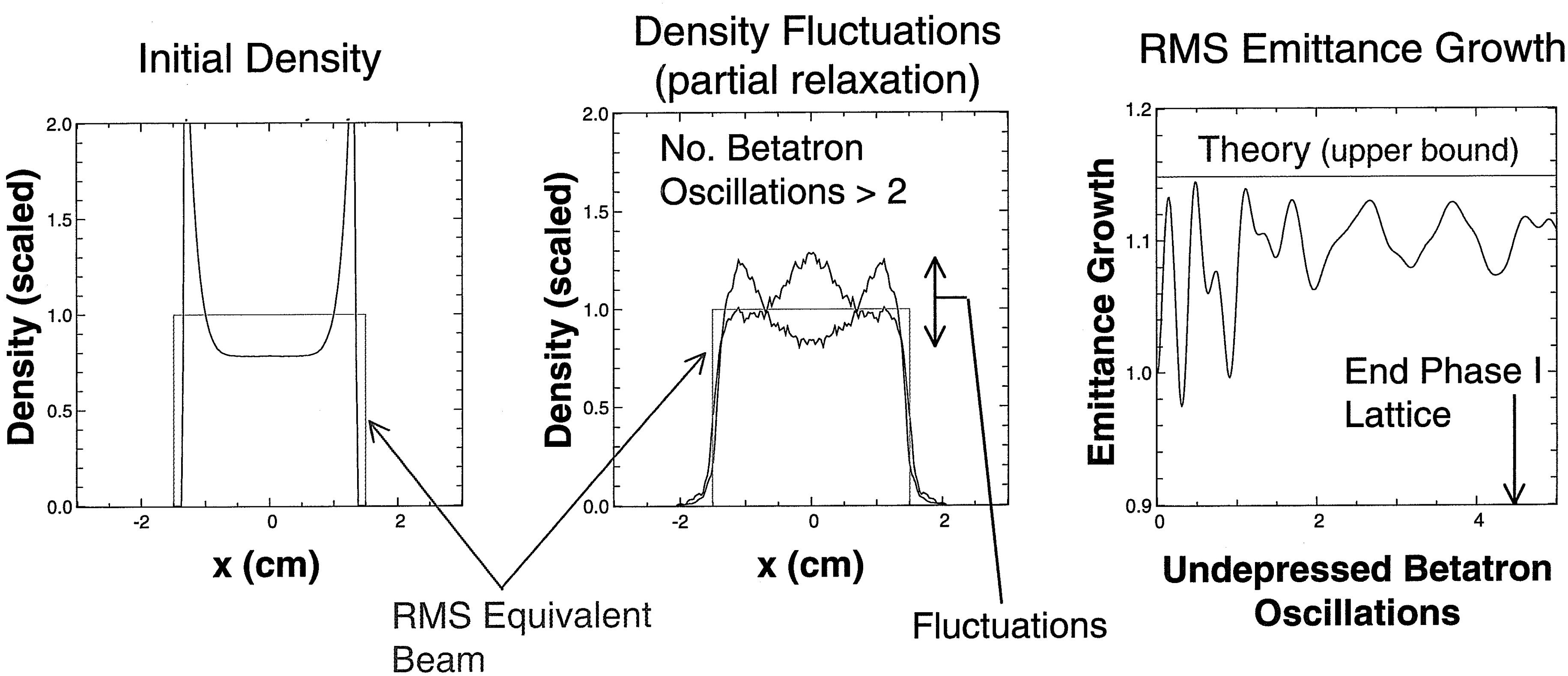
High resolution mid-pulse simulations carried out for an unbunched beam centered on the machine.

- 50-200 grids across characteristic beam radius
- 50-400 leap-frog steps per lattice period, up to 100 period advances
- 100-1000 particles per grid cell
- Round, cylindrical beam pipe > 2x beam edge to reduce images

Perturbations launched by initial distribution nonuniformities can phase-mix to a more uniform profile with increased emittance

Mode spectrum launched can undergo a rapid cascade, settling to a smaller amplitude and lower order distortion

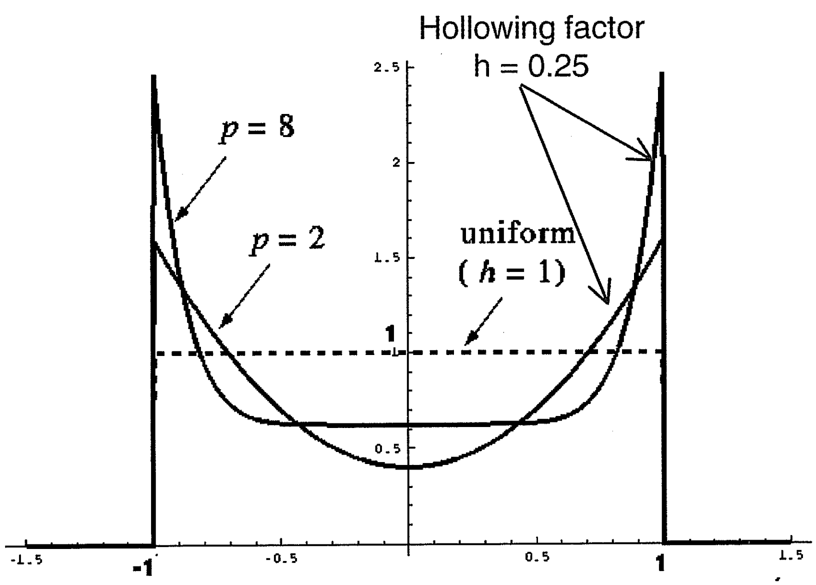
- **Approximate conservation constraints employed to bound emittance increases resulting from full relaxation to a uniform profile [Lund, Lee, and Barnard, Proc. Linac 2000, pg. 290]**
- **How will such evolutions influence the range and interpretation of measurements**



Analytic theory has been used to parametrically bound emittance growth due to the relaxation of space-charge nonunifomities

Approximate conservation constraints can be employed to estimate maximal emittance increases resulting from the relaxation of an initial nonuniform density profile to a final, uniform profile [Lund, Lee, and Barnard, Proceedings Linac 2000, Monterey, CA, pg. 290]

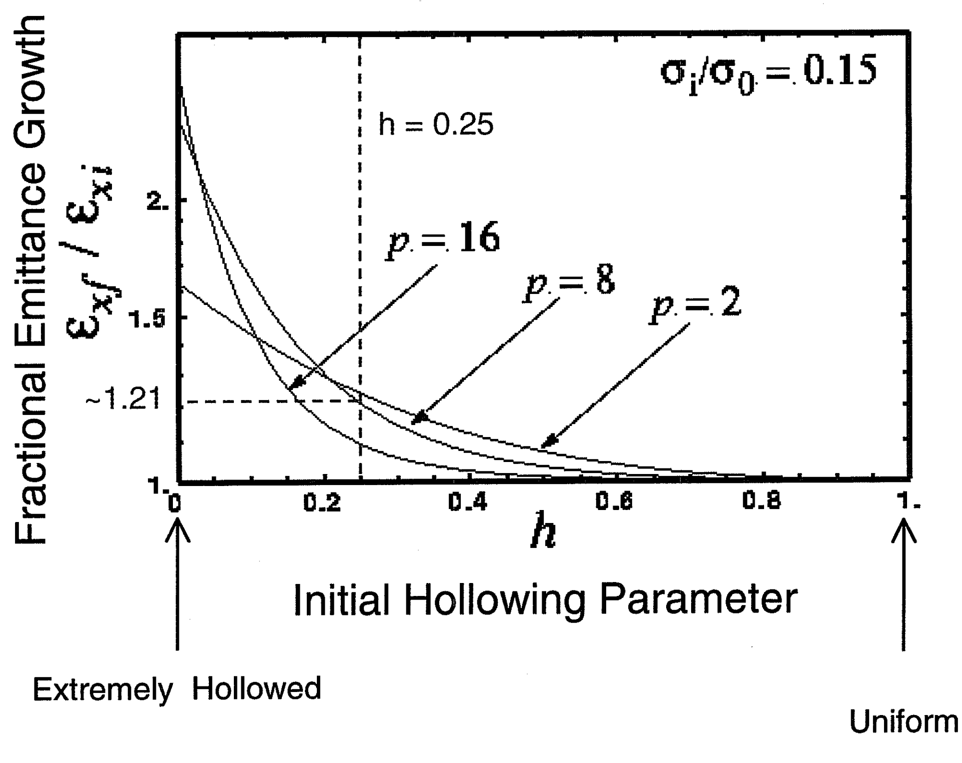
Initial Density



hollowing ~ r^p

h = ratio min to max density

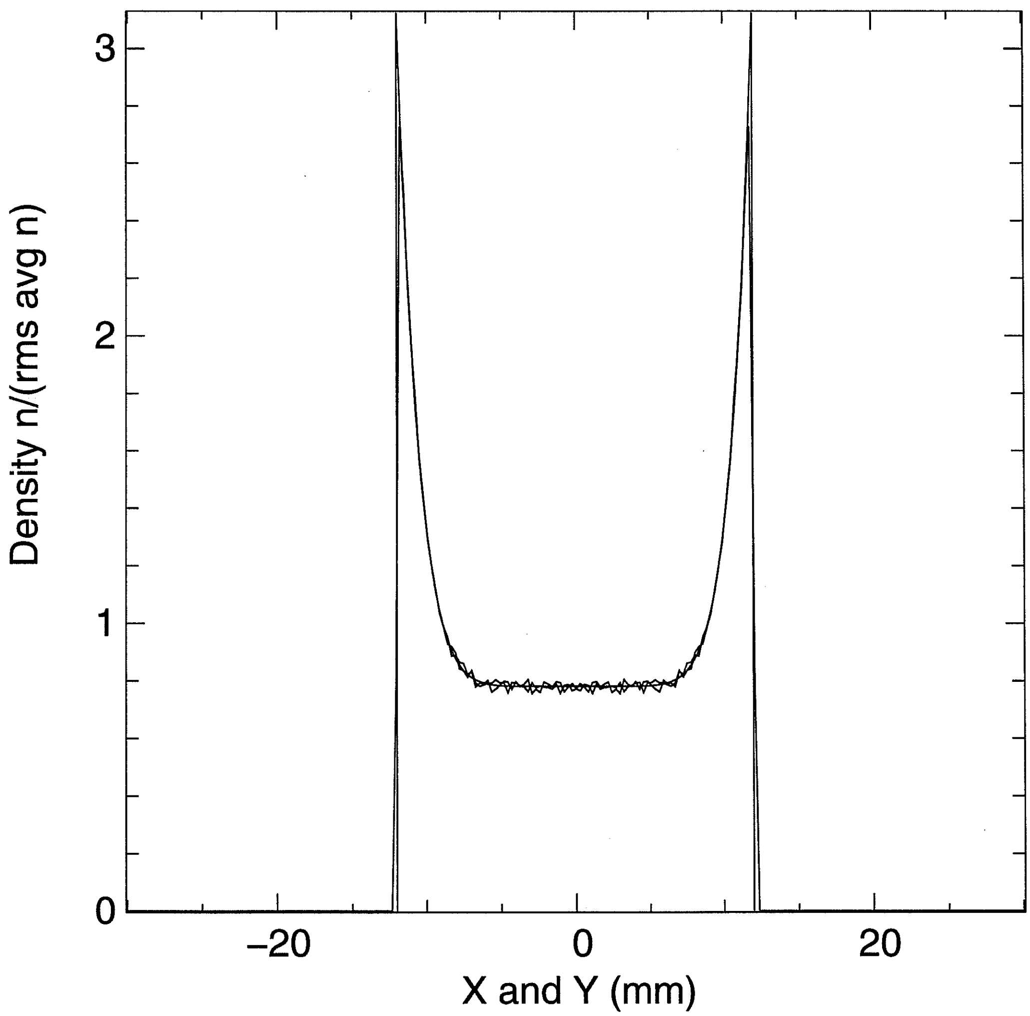
Emittance Growth on Relaxation



Continuous Focusing Simulations

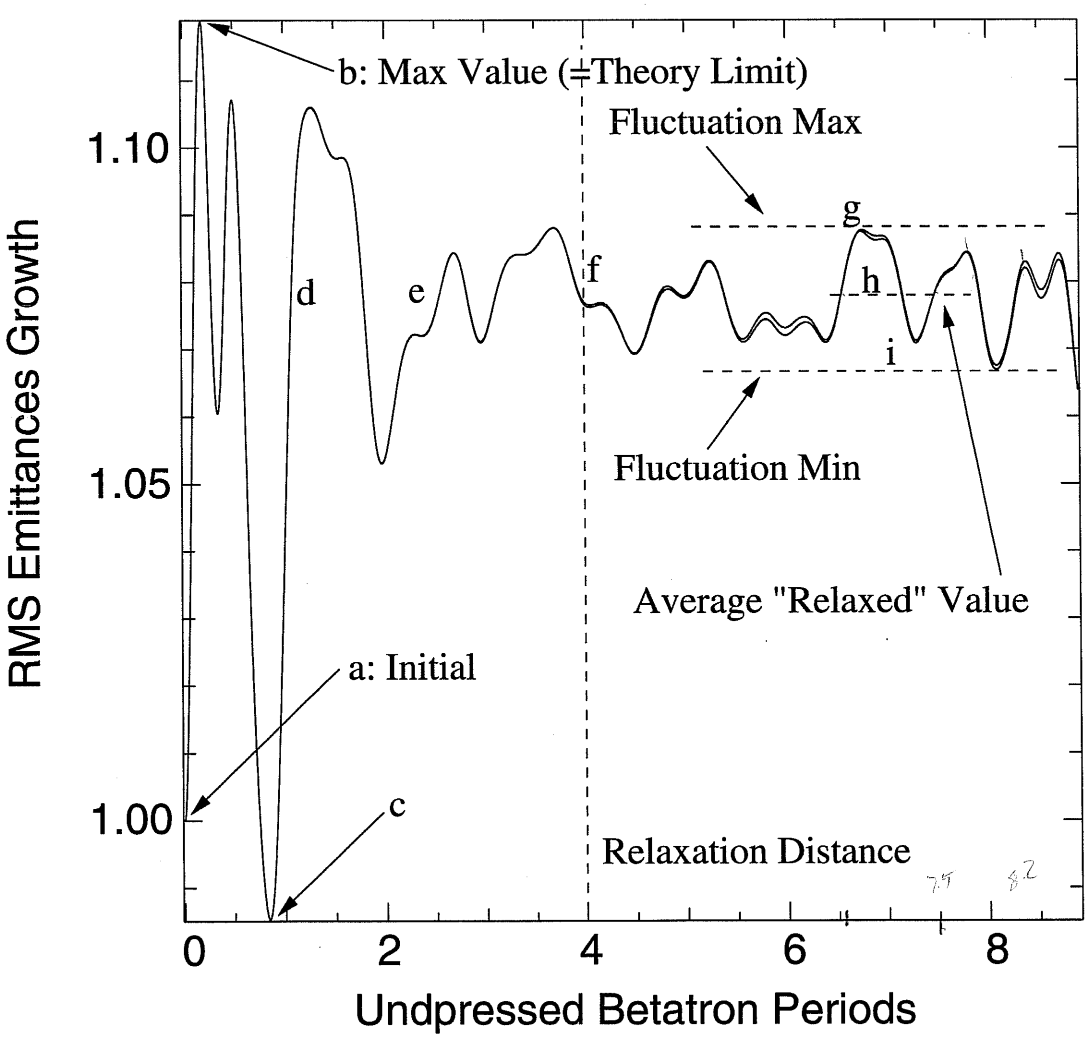
Inital Density Profile

Density: $h = 0.25$ $p = 8$



Continuous Focusing Simulations
rms Emittance Evolution

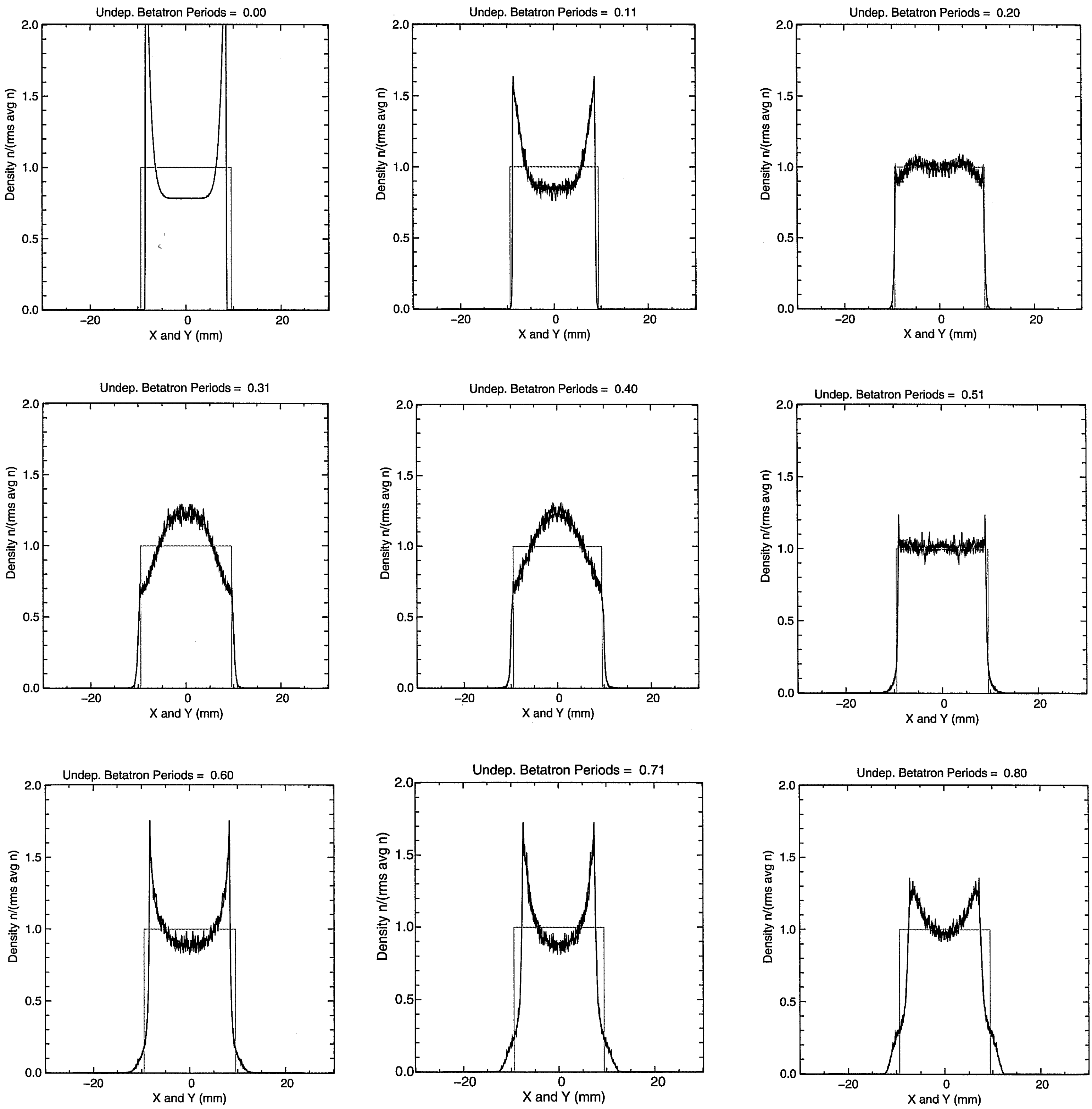
Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Continuous Focusing Simulations

Transient Density Profile Evolution

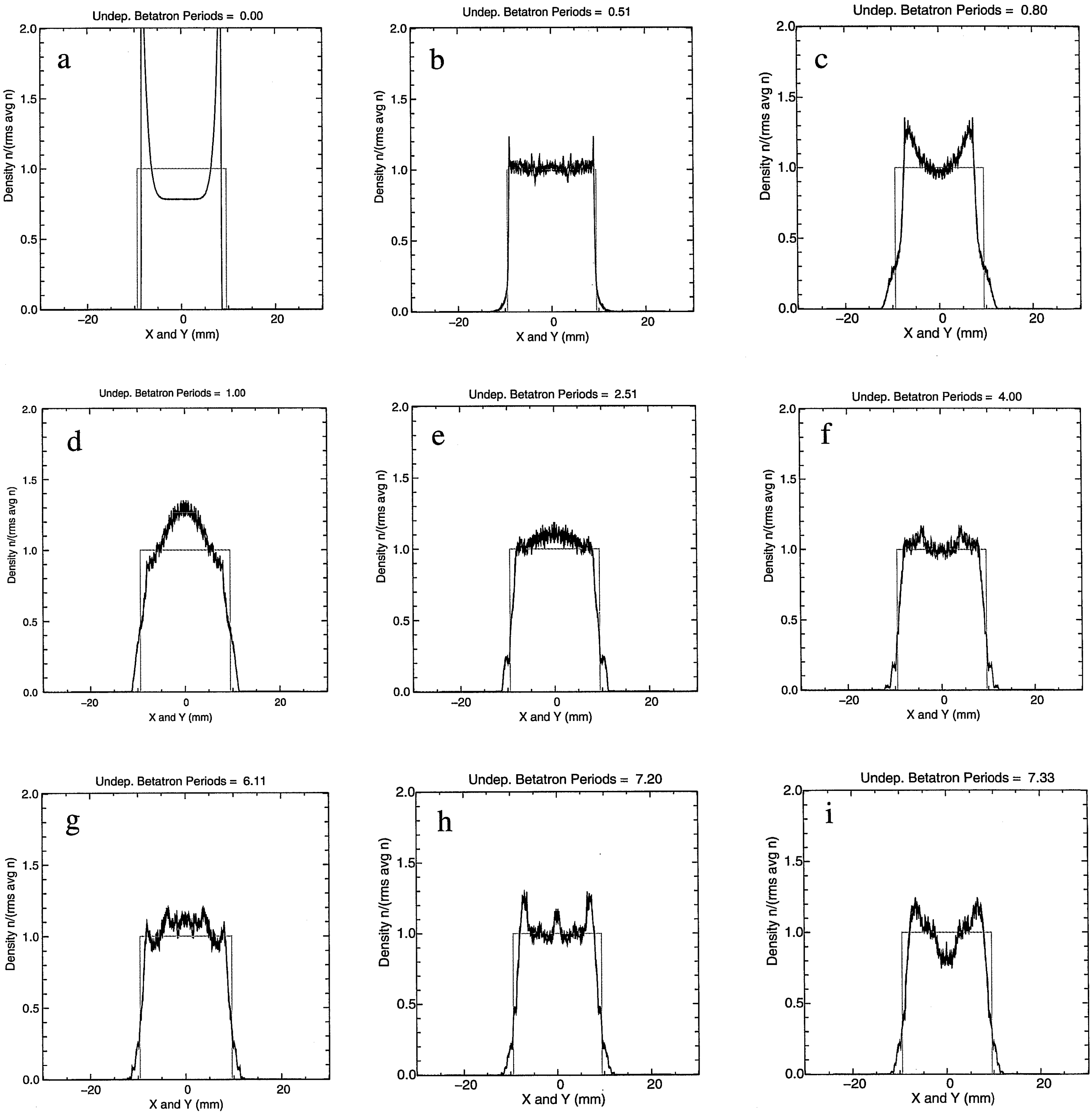
Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Continuous Focusing Simulations

Density Profile Evolution -- Initial and Saturated

Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Continuous Focusing Simulations
Summary of Results

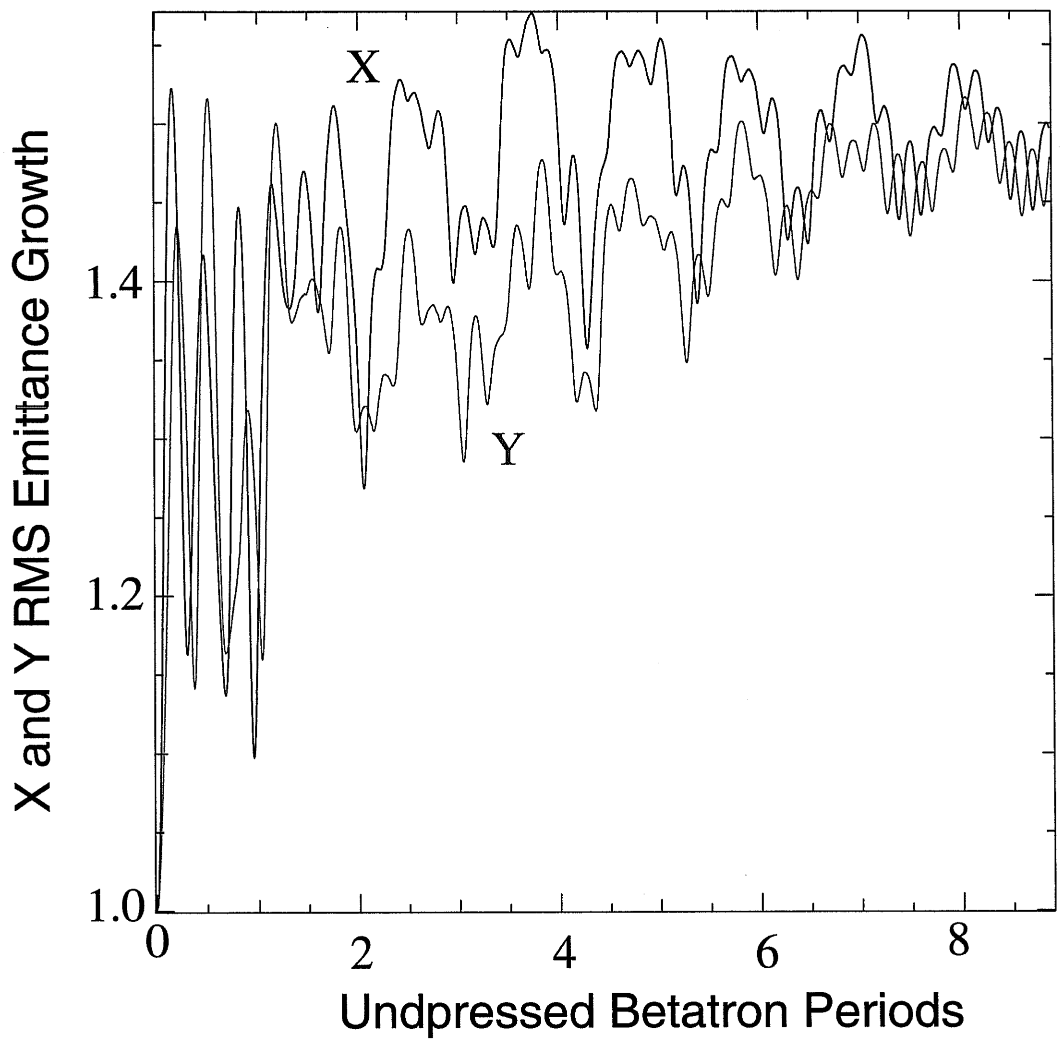
Simulated Emittance Growth Values:
Avg. Relaxed (Peak, Min. –– Max. Fluctuations)

Initial Beam					Relaxed and Transient Beam		
	Density		Temp.		Emittance Growth $\varepsilon_x/\varepsilon_{xi}$		Undep. Betatron
σ_i/σ_0	h	p	h	p	Theory	Simulation	Periods to Relax
0.1	0.25	4	1	arb.	1.57	1.42 (1.57, 1.31–1.52)	3.5
			∞	2		1.45 (1.57, 1.38–1.52)	3.0
			0.5			1.41 (1.57, 1.30–1.52)	3.0
	0.25	8	1	arb.	1.43	1.33 (1.43, 1.28–1.38)	3.5
			∞	2		1.35 (1.43, 1.30–1.40)	4.5
			0.5			1.32 (1.43, 1.26–1.38)	4.0
0.20	0.25	4	1	arb.	1.17	1.11 (1.16, 1.09–1.13)	4.5
			∞	2		1.12 (1.16, 1.10–1.13)	3.0
			0.5			1.11 (1.16, 1.09–1.13)	4.0
	0.25	8	1	arb.	1.12	1.08 (1.12, 1.06–1.09)	5.5
			∞	2		1.08 (1.12, 1.07–1.09)	4.0
			0.5			1.08 (1.12, 1.06–1.09)	4.5
0.30	0.25	4	1	arb.	1.073	1.037 (1.067, 1.035–1.040)	4.0
			∞	2		1.030 (1.067, 1.025–1.035)	3.5
			0.5			1.034 (1.067, 1.030–1.037)	4.0

Alternating Gradient Focusing Simulations

rms Emittance Evolution

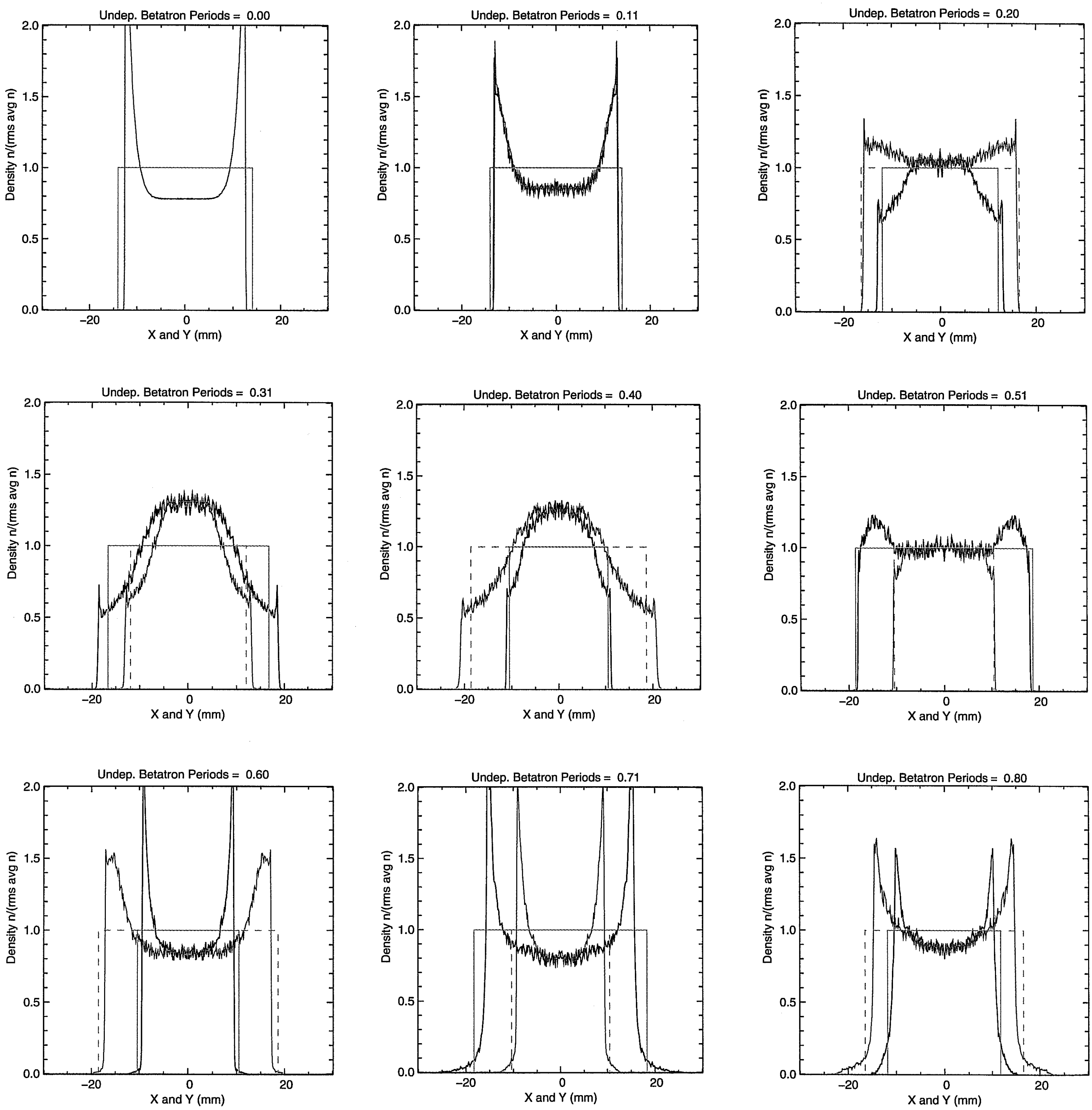
Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Alternating Gradient Focusing Simulations

Transient Density Profile Evolution

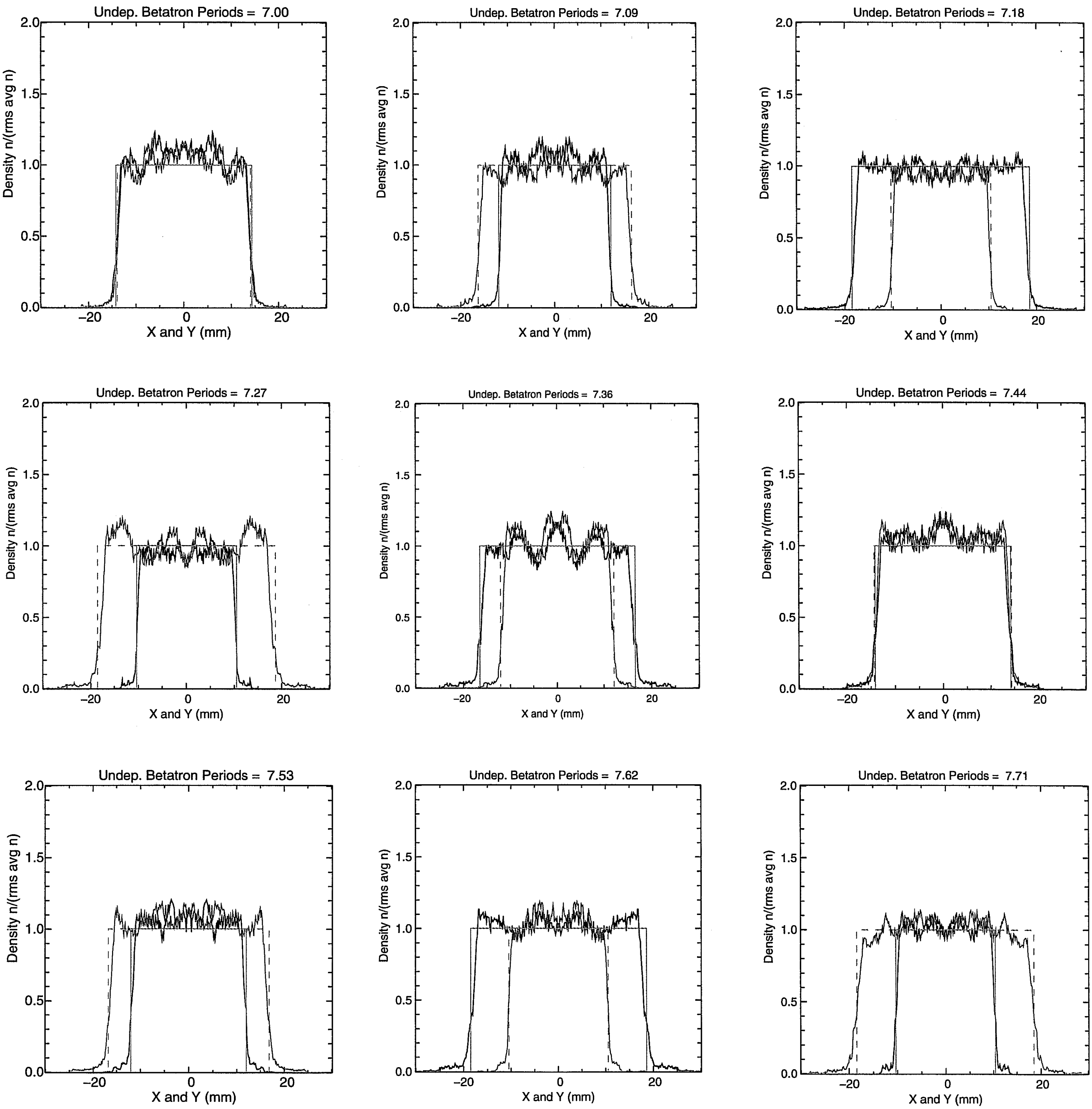
Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Alternating Gradient Focusing Simulations

Density Profile Evolution -- Initial and Saturated

Density: h = 0.25 p = 8
Temp: h = infinity p = 2



Alternating Gradient Focusing Simulations

Summary of Results

Simulated Emittance Growth Values:

Avg. Relaxed (Peak, Min. –– Max. Fluctuations)

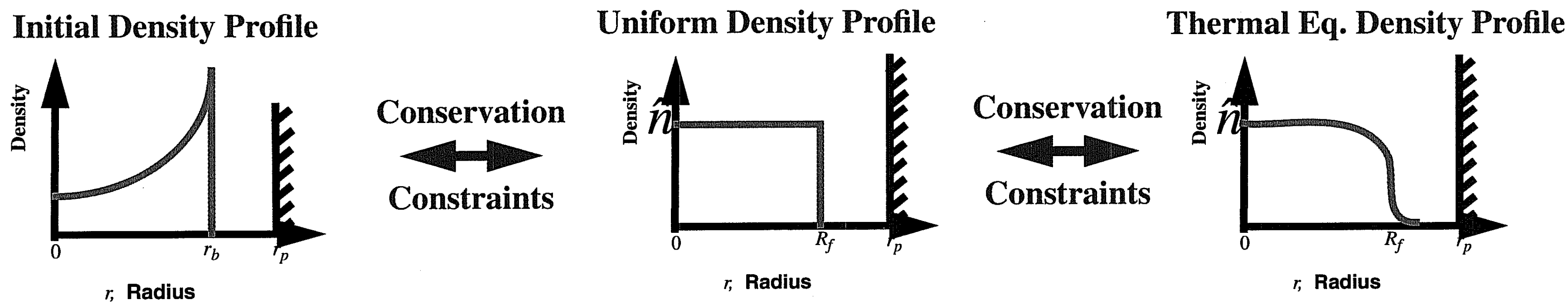
entries: [x–growth, y–growth]

Initial Beam					Relaxed and Transient Beam		
	Density		Temp.		Emittance Growth $[\varepsilon_x/\varepsilon_{xi}, \varepsilon_y/\varepsilon_{yi}]$		Undep. Betatron
σ_i/σ_0	h	p	h	p	Theory	Simulation	Periods to Relax
0.1	0.25	4	1	arb.	1.57	[1.60,1.56] ([1.72, 1.69], [1.56, 1.53]–[1.63, 1.59])	[6.5, 7.5]
			∞	2		[1.63,1.64] ([1.71, 1.70], [1.57, 1.60]–[1.68, 1.68])	[6.5, 6.5]
			0.5			[1.62,1.57] ([1.71, 1.68], [1.58, 1.52]–[1.66, 1.62])	[6.5, 7.5]

Sensitivity of Results to the Final Distribution (1)

Real beams would not be expected to relax to a final state with uniform space-charge but rather a smooth distribution that is monotonic decreasing in the single-particle transverse energy

- **Relaxation to other smooth distribution can be analyzed (for theoretical convenience) as a cascade process with conservation laws**

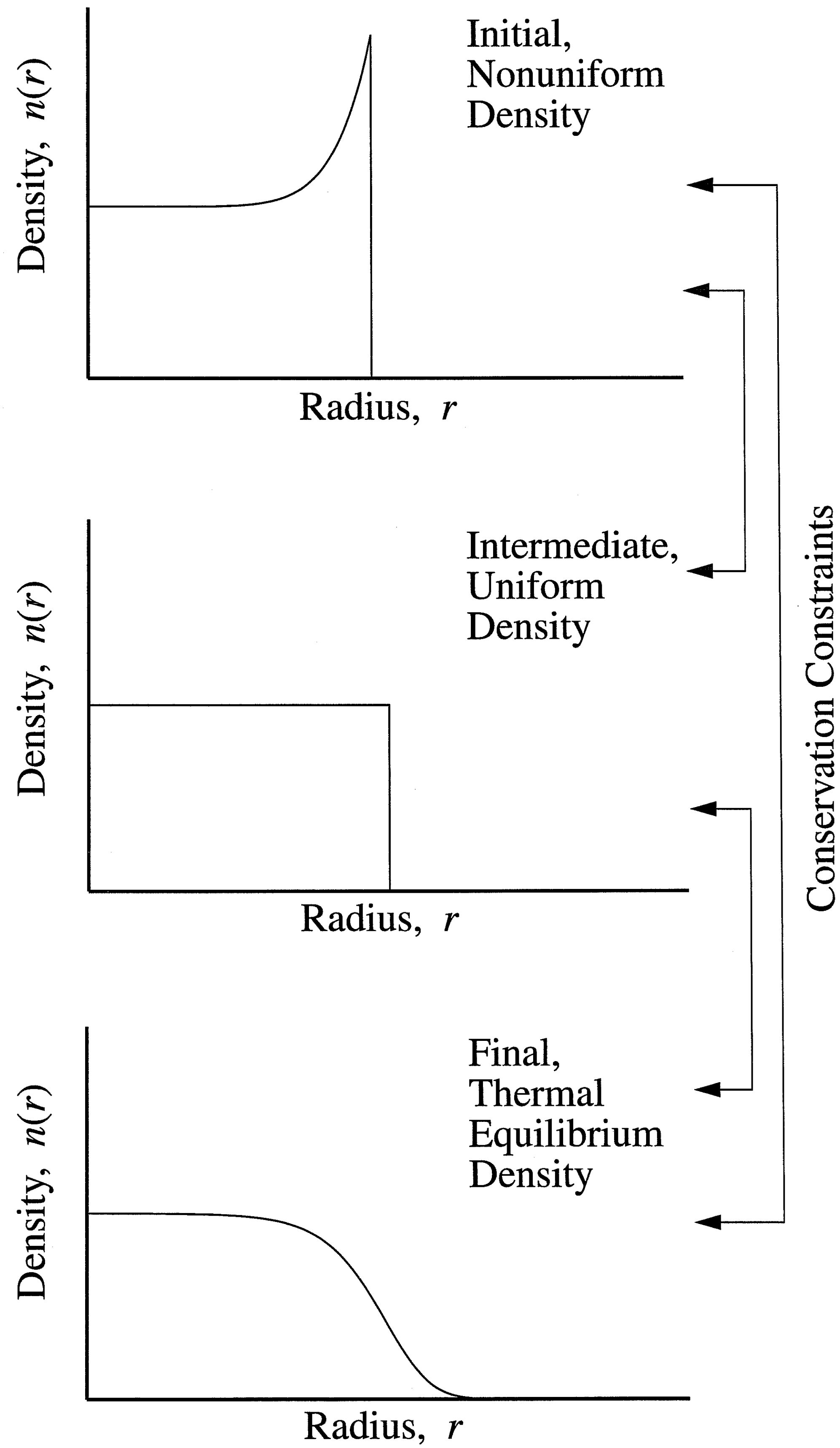


Method has been applied to show that the change in emittance on relaxation from a uniform to a smooth thermal equilibrium distribution is very small

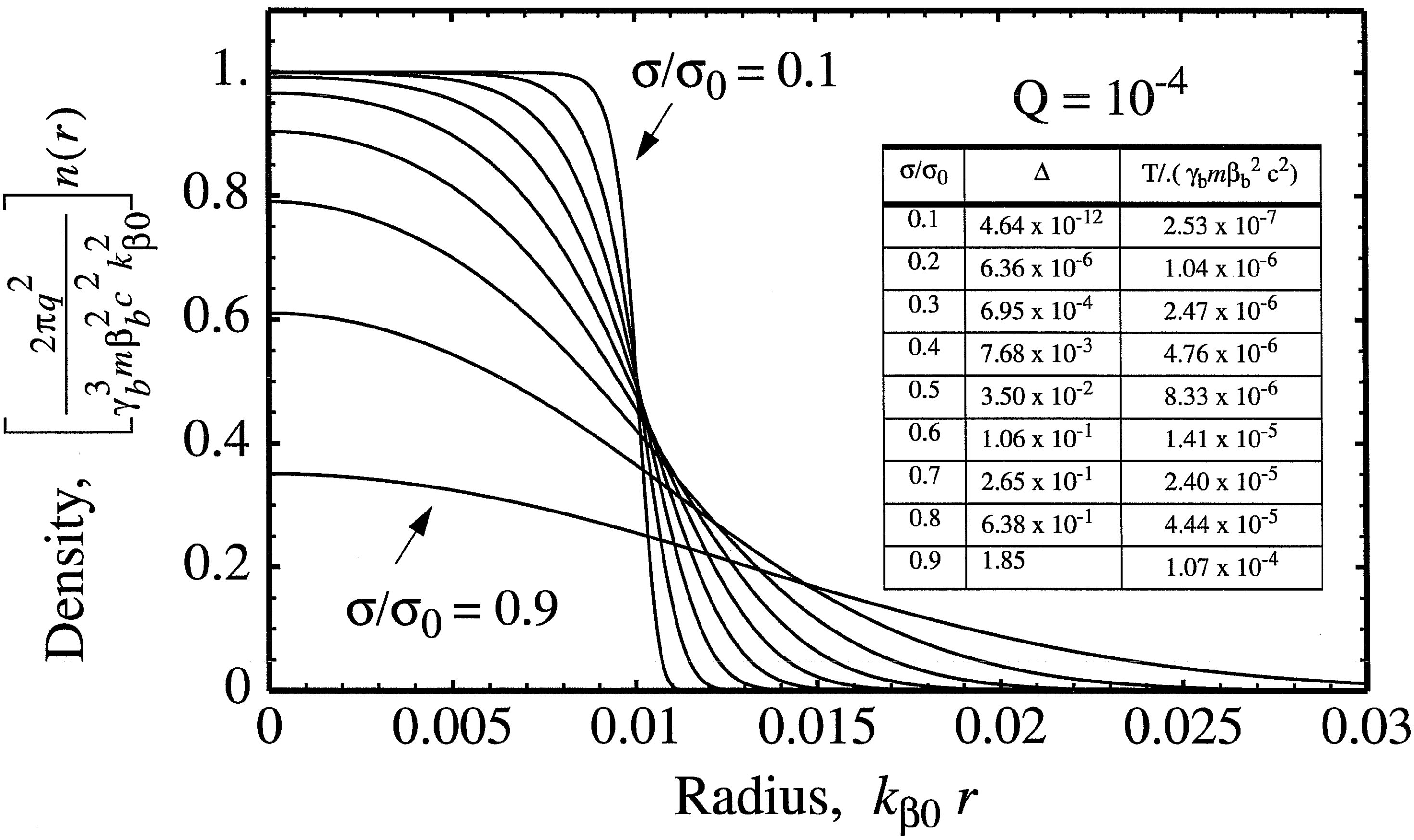
[S.M. Lund, J.J. Barnard, and J.M. Miller, "On the Relaxation of Semi-Gaussian and K-V Beams to Thermal Equilibrium," Proceedings of the 1995 Particle Accelerator Conference, Dallas, TX, May 1-5, 1995, p. 3280]

- **Only small differences are expected between thermal and other smooth distributions in the space-charge dominated regime**

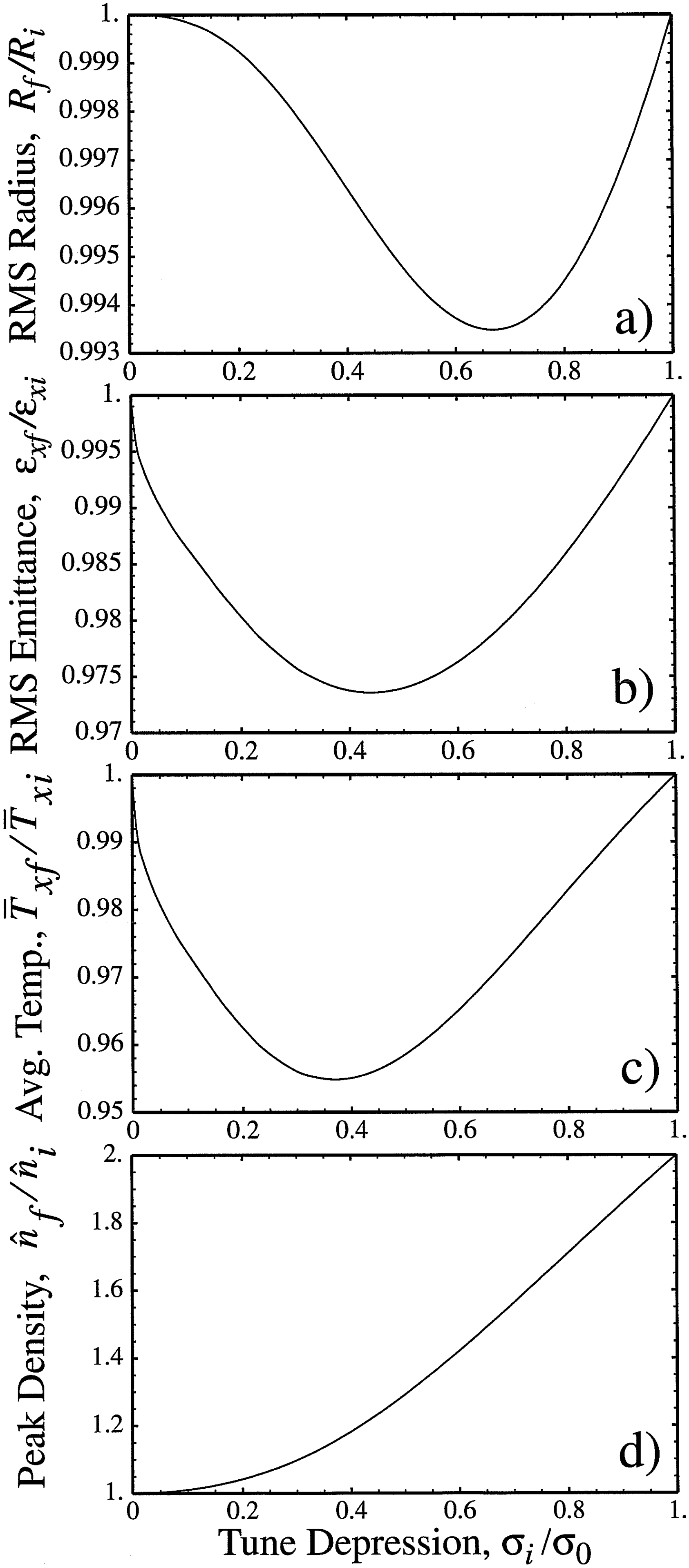
For purposes of quantifying any significant emittance increases, results presented here should accurately model a wide variety of more physical choices in final distribution



Initial,
Nonuniform
Density
Density, $n(r)$
Radius, r
Intermediate,
Uniform
Density
Density, $n(r)$
Radius, r
Final,
Thermal
Equilibrium
Density
Density, $n(r)$
Radius, r
Conservation Constraints



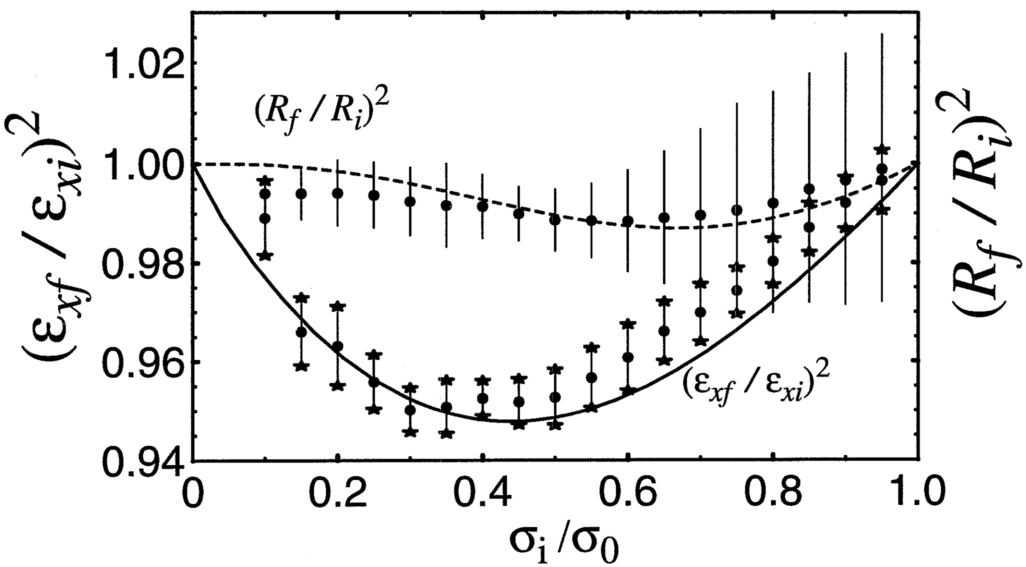
σ/σ_0	Δ	$T/.(\gamma_b m \beta_b^2 c^2)$
0.1	4.64×10^{-12}	2.53×10^{-7}
0.2	6.36×10^{-6}	1.04×10^{-6}
0.3	6.95×10^{-4}	2.47×10^{-6}
0.4	7.68×10^{-3}	4.76×10^{-6}
0.5	3.50×10^{-2}	8.33×10^{-6}
0.6	1.06×10^{-1}	1.41×10^{-5}
0.7	2.65×10^{-1}	2.40×10^{-5}
0.8	6.38×10^{-1}	4.44×10^{-5}
0.9	1.85	1.07×10^{-4}

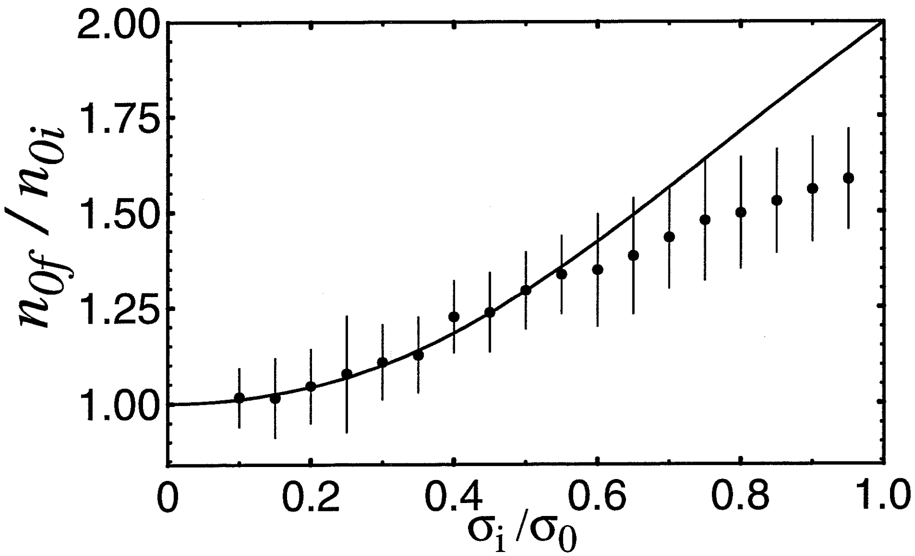


RMS Radius, R_f/R_i
a)
RMS Emittance, $\varepsilon_{xf}/\varepsilon_{xi}$
b)
Avg. Temp., $\bar{T}_{xf}/\bar{T}_{xi}$
c)
Peak Density, $\hat{n}_f/\hat{n}_i$
d)
Tune Depression, σ_i/σ_0

Sensitivity of Results to the Final Distribution (2)

Solid curves quantify small decreases in rms edge measures for the matched beam radius and emittance that result on the relaxation of an initial, rms matched KV or Semi-Gaussian distribution to thermal equilibrium and dots are the results of PIC simulations (dashed lines indicate fluctuations)





Why Large Amplitude Perturbations can be Thermalized in Intense Beams with Small Emittance Growth (1)

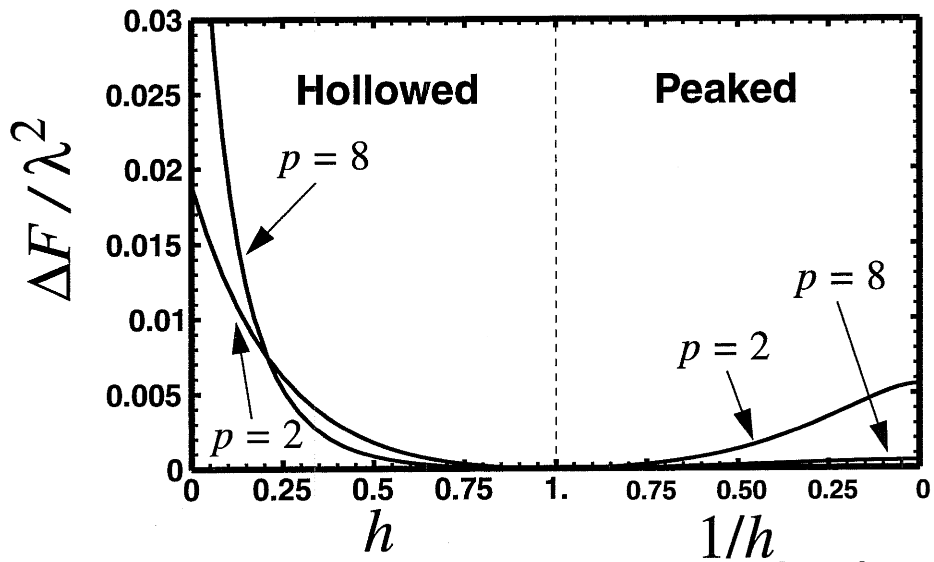
For intense beams even a large relative changes in emittance leads to small changes in rms beam radius

$$\frac{d^2R}{ds^2} + \boxed{k_{\beta 0}^2 R - \frac{Q}{R}} - \frac{\varepsilon_x^2}{R^3} = 0 \qquad \Rightarrow \qquad R_f \approx R_i \approx \frac{\sqrt{Q}}{k_{\beta 0}}$$

Large Terms ~ Balance

For fixed charge ($\lambda_i = \lambda_f$) and rms radius ($R_i = R_f$) the difference in electrostatic field energy (ΔF) between an initial hollowed or peaked density profile and a final uniform density profile can be calculated as

$$\Delta F \equiv W_i - W_f = \lambda^2 \left\{ -\frac{p(1-h)[4+p+(3+p)ph]}{(p+2)(p+4)(2+ph)^2} + \frac{1}{2}\log\left[\frac{(p+4)(ph+2)}{(p+2)(ph+4)}\right] \right\}$$



The free energy ΔF is relatively small even for large hollowing (h -> 0) and peaking ($1/h$ -> 0) factors accounting for the modest emittance growth

Why Large Amplitude Perturbations can be Thermalized in Intense Beams with Small Emittance Growth (2)

For general distributions the free electrostatic energy F of an arbitrary (nonuniform) initial distribution at fixed charge (λ) and rms radius (R) can be analyzed using variational methods:

$$F[\phi] = \int\left\{\frac{|\nabla\phi|^2}{8\pi} - \mu_1 r^2 n - \mu_2 n\right\}da \qquad \nabla^2\phi = -4\pi\rho$$

$$\mu_1, \mu_2 = constants$$

$$\delta F[\phi] = \int\{q\phi - \mu_1 r^2 - \mu_2\}\delta n da + \int\frac{|\nabla\delta\phi|^2}{8\pi}da \quad \text{(to Arbitrary Order)}$$

It follows that:

- **Constrained extrema of F satisfy (in beam)**

  $$q\phi = \mu_1 r^2 + \mu_2$$

  -- Only solution consistent with this is a uniform density beam

- **Variations about the uniform density extremum satisfy $\delta F > 0$ and are second order in $\delta\phi$**

From these results one might expect the result of modest emittance growth to be much more general than the specific choice of parabolically hollowed and peaked initial density profiles employed for convenience here

Conclusions

PIC Simulations have been employed to show that beams with high space-charge intensity transported in linear applied focusing channels can withstand large initial space-charge nonuniformities.

- Perturbations typically launch a broad spectrum of collective modes internal to the beam
- Phase mixing and nonlinear interactions quickly drive the beam to a relaxed state with a more uniform density profile and lower-order mode fluctuations

Large initial perturbations tolerable even with high space-charge intensity.

- Emittance growth and halo minimal
- Beam envelope match and control maintained

Simulations are consistent with earlier analytical theory based on conservation constraints and also provide information on relaxation times and residual fluctuations that are not obtained in the theory.

- Results appear to apply to both alternating gradient and continuous focusing

Future and Ongoing

Combined effects of beam mismatch are being explored.

- Theory (done) indicates little change in results if mismatch energy not damped
- Simulations are underway to test and verify results

References and Acknowledgments

Much of the work builds on earlier work employing energy conservation by M. Reiser and others. See for example:

M. Reiser, *Theory and Design of Charged Particle Beams*, Wiley, 1994
R.C. Davidson, *Physics of Noneutral Plasmas*, Addison Wesley, 1990

Theories of collective modes internal to a space-charge dominated beam have been developed in:

S.M. Lund and R.C. Davidson, Phys. Plasmas, **5**, 3028 (1998)

Theory has been extended considerably to include mismatch etc. beyond what is presented here:

S.M. Lund, R.C. Davidson, J.J. Barnard, E.P. Lee, Beam Emittance Growth from the Collective Relaxation of Space-Charge Nonuniformities, To be Submitted for Publication.

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Mailing List

Copies of the full paper will be distributed (email preferred). Contact smlund@lbl.gov or sign up below:

lwu2@uiuc.edu

tkikuchi@cc.utsunomiya-u.ac.jp

benedetti @ bo. infn. it

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